SCIENTIFIC OASIS

Decision Making: Applications in Management and Engineering

Journal homepage: www.dmame-journal.org
ISSN: 2560-6018, eISSN: 2620-0104

Performance Evaluation and Lockdown Decisions of the UK Healthcare System in Dealing with COVID-19: a Novel Unbiased MCDM Score Decomposition into Latent Vagueness and Randomness Components

Peter Wanke[1], Jorge Antunes[1], Yong Tan[2], Seyyed Ahmad Edalatpanah[3,*]

1 COPPEAD Graduate Business School, Federal University of Rio de Janeiro, Rua Paschoal Lemme, 355 Rio de Janeiro CEP: 21949-900, Brazil
2 School of Management, University of Bradford, Bradford, West Yorkshire BD7 1DP, UK
3 Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Mazandaran, Iran

ARTICLE INFO

Article history:
Received 14 November 2023
Received in revised form 13 December 2023
Accepted 16 December 2023
Available online 15 January 2024

Keywords: Lockdown drivers; Multiple-Criteria Decision Making; Latent Vagueness and Randomness Components; COVID-19; Unbiased scores.

ABSTRACT

This paper examines the performance evaluation and lockdown drivers of the UK's NHS (National Health Service) during the COVID-19 pandemic. The study aims to enhance the NHS's response to future health crises and guide government lockdown decisions. Lockdown drivers encompass vital resources like beds, ventilators, patients, and staff. A three-stage MCDM (Multiple-Criteria Decision Making) approach is employed to analyze performance scores. First, partial utility functions or partial distances are computed using COPRAS (Complex Proportional Assessment) and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), respectively. Second, the Latent Vagueness and Randomness Components (LAVRA) method filters unbiased performance scores from uncertain components. Third, a bootstrapped neural network regression classifies lockdown drivers based on performance, deaths, and geographic regions. Crucial drivers relate to ventilated bed availability, while less critical ones include staff absence due to COVID-19 and a high admission rate of elderly inpatients. The results indicate performance scores range from 0.65 to 0.75 using TOPSIS, while COPRAS analysis significantly reduces the scores. Lockdown decisions are influenced by geographic regions, death tolls, and unbiased hospital performance scores.

1. Introduction

The novel coronavirus is named Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2), and the associated disease is called COVID-19 [1]. This virus has the characteristics of a very fast spread rate and also has a devastating effect on people's lives [2]. According to the data provided by worldometer.info, as of the 18th of February 2022, a total of 225 countries in the world were affected by this virus. In terms of the number of positive cases since the outbreak, there have been more than 79 million in the United States, which was ranked as the number one country in the world, followed

* *Corresponding author.*
E-mail address: saedalatpanah@gmail.com

https://doi.org/10.31181/dmame7120241041

by Brazil, India, Brazil, and France. The UK was the fifth country in the world in terms of the number of accumulated positive cases. The data show that the number of deaths from COVID-19 follows a similar pattern as the positive cases among the countries in the world; the USA has the largest number of accumulated deaths, exceeding more than 95 thousand, followed by Brazil, India, and Russia. The UK was ranked 7th, following Mexico and Peru, in terms of the number of accumulated deaths, exceeding more than 16 thousand [3].

Investigating the issue of NHS performance, particularly during the pandemic, will not only provide policy implications to improve its capability in normal scenarios but will also offer useful guidance on how to handle similar pandemics in the future. Reviewing the literature reveals that very few efforts have been made in this area to evaluate the performance of the NHS during the pandemic. The tools that can be used to measure performance can be mainly divided into three groups, among others: production functions, non-parametric data envelopment analysis, as well as clinical or accounting indicators. Although some studies apply the multiple-criteria decision-making method in evaluating various issues in the NHS [4,5], no clear effort has yet been made to evaluate the NHS's performance during the pandemic. We contribute to the literature by proposing an innovative MCDM to evaluate the performance of the NHS during the pandemic, and lockdown drivers are also evaluated using a bootstrapped neural network regression.

Despite ongoing advancements in healthcare research, the field of Multiple Criteria Decision Making (MCDM) is continuously expanding, with various approaches being continually developed and integrated to investigate the inherent epistemic uncertainty in ranking alternatives and determining criteria weights for performance measurement [6]. This study, therefore, adds to the existing body of MCDM literature by concurrently examining the fundamental concepts of "utility functions" (utilizing COPRAS - Complex Proportional Assessment) and "distance to ideal solutions" (employing TOPSIS - Technique for Order Preference by Similarity to Ideal Solution) in identifying potential measures for healthcare performance (stage 1). An additional distinctive facet of this research pertains to the calculation of unbiased performance scores through the decomposition of the latent vagueness and randomness components of scores initially computed using COPRAS and TOPSIS (stage 2). By breaking down distinct pieces of information used to measure healthcare performance into their main unbiased constituents, this process offers a clear understanding of the cause-effect relationships between lockdown decisions and performance estimates calculated under the key concepts of "utility functions" and "ideal solutions," along with other contributing factors, through classificatory neural networks (stage 3).

It is noteworthy that non-parametric performance measurement approaches such as DEA (Data Envelopment Analysis) have been widely used in healthcare research [7]. With growing usage, the healthcare research stream has also witnessed the emergence of MCDM for performance measurement, where ELECTRE (ELimination Et Choice Translating REality) has been a common approach [8]. This method was first proposed by Roy [9] to help in deciding on new activities by using a weighted sum technique. There are two main parts to an ELECTRE application: first, the construction of one or several outranking relations, which aims at comparing each pair of actions comprehensively; second, an exploitation procedure that elaborates on the recommendations obtained in the first phase. The nature of the recommendation depends on the problem being addressed: choosing, ranking, or sorting.

Differently from previous applications in healthcare, however, the novel approach developed in this paper was designed to handle the impacts of the unique characteristics of the COVID-19 pandemic over datasets, criteria adopted, and the reliability of weights assigned. First, due to the unprecedented nature of this crisis, little was known whatsoever at the time of data collection concerning the random nature of each criterion collected, whether it differed or not from previous studies' assumptions, based on non-parametric approaches. Second, the limited time to deploy physical and human resources to the field also implied a limited amount of time to collect detailed information on distinct types of resources, staff expertise, and demographic characteristics of the population served. These also imposed epistemic limitations as regards the list of criteria used in terms of the lack of comparability with previous similar studies, if any. Third, based on the lack of information on the random nature and scope of criteria collected, the most appropriate MCDM approach as regards the performance measurement was not whatsoever clear to help in decision-making. For instance, TOPSIS is preferred when positive and negative ideals are known, while, on the other hand, COPRAS should be adopted when decision-makers are convinced to measure the utility functions of the resources deployed [10]. It should be noted that, differently from ELECTRE, where superiority relationships are determined by making binary comparisons between the alternatives provided, under TOPSIS and COPRAS, instead of binary comparisons, continuous distances among alternatives are computed in terms of the positive/negative ideal solutions and their utility functions, respectively [11].

Hence, as long as none of these issues was completely mapped and by no means comparable to any similar situation studied in the recent past in healthcare systems, the decomposition of the underlying measurement uncertainty into its latent probabilities (randomness) and possibilities (fuzziness) under continuous distances and functions was deemed necessary, taking simultaneously two cornerstone approaches for measuring performance: ideal solutions and utility functions. The modeling choice of decomposition of uncertainty into its latent components, of course, brings the need to not only analyze the problem stochastically but also to select the proper regression tools to unveil the non-linear relationships that may eventually arise among the variables under study. One should also note that the use of stochastic modeling for measuring performance and its determinants also addresses two of the major criticisms regarding DEA modeling as mentioned by Simar and Wilson [12]: (i) the inexistence of a proper inference approach with respect to performance scores; (ii) the intrinsic bias related to separability issues as regards productive frontiers and their determinants, which tend to be endogenous.

Our results show that the performance scores of the UK NHS during the COVID-19 period were between 0.65-0.75, as estimated from the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS); however, the performance scores were significantly lower, as reflected by the Complex Proportional Assessment (COPRAS). This result indicates that the argument regarding the level of performance in the UK NHS will depend on the data used as well as the methods adopted in the data analysis. In terms of the lockdown decisions, our findings show that they are influenced by geographic regions, death tolls, and unbiased hospital performance scores.

The rest of this paper is structured as follows. Section 2 focuses on the contextual setting related to the challenges faced by the UK NHS during the COVID-19 pandemic. Section 3 focuses on the literature review and the methodological research gap. Section 4 depicts the novel three-stage stochastic-fuzzy-neural MCDM approach. Data analysis and discussion of results are given in Section 5. The paper ends in Section 6 with the conclusions of the study and its managerial and policy implications provided for the UK NHS.

2. COVID-19 in the UK

The UK has experienced several stages and implemented a series of measures and policies to deal with COVID-19. On the 23rd of March 2020, the Prime Minister announced the first national lockdown lasting for nearly two months. The conditional lift of the lockdown was announced on the

10th of May, after which people who could not work from home were instructed to return to the workplace without the use of public transport. The COVID-19 situation improved in the summer of 2020, during which it was observed that daily positive cases, as well as daily deaths, experienced a decline. However, there was another upsurge in September after several measures were implemented to ease restrictions in June, July, and August 2020. These measures included the reopening of non-essential shops and schools in England in June, the reopening of indoor theaters, bowling alleys, and soft play in August, as well as the implementation of the rule of six in September. The UK entered the second national lockdown on the 5th of November 2020. On the 24th of November, the Prime Minister announced that up to three households would be able to meet over five days between the 23rd and 27th of December 2020, and plans were made to reopen schools after Christmas. The UK went into a third national lockdown on the 6th of January 2021. In February 2021, the UK introduced hotel quarantine for travelers arriving in England from 33 high-risk countries.

From March 2021 onward, the COVID situation in the UK gradually improved as vaccinations were received by the people. The easing of restrictions had been planned by the UK government in four phases. Starting from the 29th of March 2021, outdoor gatherings of up to 6 people or 2 households would be allowed, with outdoor sports facilities to be reopened, but international travel remained banned. People were encouraged to continue working from home where possible. The second phase started on the 12th of April when non-essential retail and personal care premises would reopen, outdoor attractions and indoor leisure facilities would be reopened, but the social contact rule would still apply, and the usage of these would only be restricted to people of their own household. Funerals could be held with a maximum of 40 people and weddings with 15 people. The third phase was planned to be carried out on the 17th of May when most social contact rules would be lifted, although indoor activities would remain restricted to a maximum of 6 people. Outdoor performances would be reopened, allowing up to 10,000 people. Indoor hospitality and entertainment venues were also supposed to reopen, allowing up to 1,000 people, and up to 30 people would be allowed to attend funerals and/or weddings. The final stage was implemented on the 19th of July when all legal limits on social contact were removed, and people's lives eventually returned to normal completely. On the 8th of December, a move to plan B was announced to deal with the spread of the Omicron variant. The COVID pass became mandatory in nightclubs on the 15th of December 2021. Table 1 shows the timeframe of COVID-19 in the UK and relevant policies.

3. Contextual Setting

The NHS represents the National Health Service, a publicly funded national healthcare system in the UK founded in 1948 [13]. The main mission is to provide healthcare services to the people in the nation irrespective of religion, gender, age, sexual orientation, race, and disability [14]. The NHS is one important component in the Department of Health, the head of which is called the Secretary of State for Health. The Department of Health is responsible for the operation and management of the department, including setting policies for the NHS, such as waiting time, funding, and staffing targets, and it directly reports to the Prime Minister [15].

Significant reforms were made to the NHS in 2012 when it was divided into a series of organizations that operate at both national and local levels. This followed the previously abolished Strategic Health Authorities and Primary Care Trusts. The Clinical Commissioning Groups are responsible for commissioning healthcare services for their local areas and are run by general practitioners, consultants, and nurses, controlling almost 2/3 of the NHS budget [16]. The Clinical Commissioning Groups are overseen by the NHS Commissioning Board, which has both regional and local offices around England [17]. The NHS Foundation Trusts provide care that the Clinical

Commissioning Groups commission, and they are mainly responsible for providing primary care services, mental health, ambulance, social care, and hospital services. The NHS Foundation Trusts are mainly made up of hospital trusts, mental health trusts, and ambulance trusts [18]. The NHS hospital trusts are foundation trusts run by local managers, staff, and members of the public and are tailored to local needs. In comparison, the ambulance trust is responsible for providing emergency access to healthcare, while the mental health trusts mainly provide health and social care services for people with mental health problems.

Table 1
Timeline of UK COVID-19 lockdowns and measures from March 2020 to December 2021

2020	
23rd March	PM announces the first lockdown in the UK, ordering people to "stay at home".
26th March	Lockdown measure leagally come into force.
10th May	PM announces a conditional plan for lifting the lockdown and says that people who cannot work from home should return to the workplace but avoid public transport.
1st June	Phased re-openning of schools in England.
15th June	Non-essential shops reopen in England.
29th June	Matt Hancock announces that the UK's first local lockdown would be applied in Leicester and parts of Leicestershire.
4th July	UK's first local lockdown comes into force in Leicester and parts of Leicestershire. More restrictions are eased in England, including reopening of pubs, restaurants, hairdressors.
14th September	"Rule of six"- indoor and outdoor social gatherings aboe six banned in England.
22nd September	PM announces new restrictions in England, including a return to working from home and 10pm curfew for hospitality sector.
5th November	Second national lockdown comes into force in England.
24th November	PM announces up to three households will be able to meet up during a five-day Christmas period between 23rd to 27th December.
2nd December	Second lockdown ends
19th December	PM announces tougher restrictions for London and Southeast England, with a new Tier 4: "Stay at Home" alert level. Christmas mixing rules tightened.
21st December	Tier 4 restructions come into force in England and Southeast England.
26th December	More areas of England enter Tier 4 restrictions
2021	
6th January	England enters third national lockdown.
15th February	Hotel quanratine for travelers arriving in England from 33 high-risk countries
March (Step 1 of lifting the lockdown)	Primary and secondary school reopen and small outdoor gathering of no more than 6 people or two househld will be allowed.
April (step 2 of lifting the lockdown)	Non-essential shos and outdoor venues reopen.
May (step 3 of lifting lockdown)	Indoor gathering of two households allowed and indoor venues reopen.
July (step 4 of lifting lockdown)	Most legal limits on social contact removed.
December	Face mask becomes compulsory in most public indoor venues and NHS COVID pass becomes mandatory in nightclubs.

NHS providers have played a vital role in dealing with COVID-19 to slow down the spread of the virus, facilitated by government policies. According to NHS England, providers have consistently treated COVID-19 patients daily, with the maximum number of admissions exceeding 4000 a day [19]. Between March 19, 2020, and February 2, 2022, 439,752 people had been discharged. Although NHS providers have done a fantastic job, their performance could be even better if some challenges could be overcome: 1) The limited availability of raw materials and labor hampers the efficient functioning of the NHS. For example, the limited number of beds, ventilators, and qualified healthcare professionals significantly reduces the capacity of NHS providers in battling COVID; 2) A significant and unexpected increase in the volume of COVID-19 cases substantially increases the demand for hospital admissions. The sudden surge in demand sometimes exceeds the capacity of NHS providers to cope; 3) Due to the priority of dealing with COVID, non-COVID patients cannot be treated swiftly. In some scenarios, this may increase the severity of non-COVID illnesses and further raise the demand for hospital beds, restricting hospitals' capacity to deal with COVID.

According to statistics from the King's Fund [20], funding for health services in England comes from the Department for Health and Social Care’s budget. The planned spending for 2021/22 was £190.3 billion, including an additional £33.8 billion in response to the COVID-19 pandemic. The main expenditures from the budget were related to staff salaries, medicines, buildings, and equipment. Comparing these statistics with those over the past 10 years, it is noted that between 2010/2011 and 2019/2020, the NHS budget increased from £119.9 billion to nearly £140 billion [21]. In terms of workforce statistics, based on the report from NHS Digital [19], there were 1,205,362 full-time equivalent staff in September 2021, increasing by 3.8% compared to the previous year. Among them, 52.8% were professionally qualified clinical staff, increasing by 3.2% compared to the previous year. According to the consolidated NHS provider accounts 2019/2020 [22], during the year 2019/2020, there were 226 NHS providers, including 130 hospital trusts, 53 mental health trusts, and 10 ambulance trusts.

The policies or measures implemented and planned by the government, as illustrated above, prioritize people’s lives and aim to reduce pressure on the National Health Service (NHS) in dealing with this public health crisis. In other words, the level of restrictions imposed by the government largely reflects the NHS's capability to slow down the virus's spread by admitting a large number of patients, especially to deal with the uncertainty derived from an unexpected increase in the number of patients at any time. A higher capability of the NHS to handle this will reduce the government's pressure to impose stricter restrictions or delay the imposition process. This is crucial for the economy and society. From the country’s perspective, the easing of restrictions would be beneficial to economic activities and promote economic growth. From the individual perspective, the easing of restrictions in terms of working patterns or social rules means that people can lead a normal life, undertake activities as usual, and communicate or socialize with others, which is crucial for mental health.

4. Literature Review

Using the County Durham and Darlington Foundation Trust, Oomman and Todd [23] investigated the influence of COVID-19 lockdown on Accident and Emergency performances. Instead of using machine learning techniques, the study employed the golden 4-hour standard and clinical quality indicators to measure performance. The former focused on assessing, treating, admitting, and discharging patients within a 4-hour window, while the latter concentrated on the amount of time patients spent in the Accident and Emergency department before moving to a ward, transferring to another hospital, or being discharged. The findings suggest that the fear of catching COVID in the hospital environment may have led to a reduction in patient numbers. Waiting time is a significant concern, especially during the COVID-19 pandemic, as managing long waiting queues for non-communicable diseases, cancers, and other conditions significantly influences the mortality and morbidity of COVID-19 patients [24].

Healthcare performance investigation does not only focus on the public sector National Health Service (NHS), but attempts have been made to investigate the sustainability of for-profit hospitals during the COVID-19 pandemic. Kruse and Jeurissen [25] argued that the financial conditions of for-profit hospitals before the outbreak of COVID-19 would determine their ability to deal with the financial shock derived from the COVID-19 crisis, and relevant reserves should be kept by hospitals to improve their ability to absorb negative shocks from the unexpected external environment. They also argued that government intervention to bail out problematic hospitals, as well as mergers and acquisitions, should be engaged in to form a more consolidated hospital market. Kuosmanen et al. [26] investigated the performance of UK NHS hospitals during the first and second waves of COVID-19. They proposed a production function of death that incorporated contextual variables under a convex quantile regression approach. To address the issue of zero observations, Heckman's two-stage approach was proposed. The results show an improvement in expected mortality in the second wave compared to the first wave, and there is a regional difference in hospital performance, but the performance gap is found to be smaller in the second wave.

Another group of studies applied the non-parametric technique to estimate the performance of the health system in dealing with COVID-19. Hamzah et al. [27] conducted network data envelopment analysis to investigate the efficiency level of Malaysia's health system in dealing with COVID. Two inputs, including the total number of confirmed cases and death cases, were considered. Recovered cases were used as outputs. Three sub-processes, including community surveillance, medical care I, and medical care II, were included in the network analysis. The findings suggest that overall inefficiency in the health system is mainly attributed to the poor performance of the medical care process. Using death rates and infection rates as undesirable outputs under non-parametric data envelopment analysis, Breitenbach et al. [28] investigated the healthcare resource efficiency of 36 countries in the world. Three inputs were considered, including the number of tests, the number of doctors and nurses, and health expenditure, with the desirable output of recovery rate included in the analysis. The findings suggest that the average efficiency of global health systems in managing the COVID-19 pandemic is low.

Another world-scale analysis of health system performance in dealing with COVID-19 was undertaken by Lupu and Tiganasu [29] under data envelopment analysis. Several inputs were considered, including COVID-19 cases, physicians, nurses, hospital beds, and health expenditure. COVID-19 death was considered as the output. The results show that the efficiency of health systems in the sample is quite low, particularly for a few European countries, including Italy, Belgium, Italy, and the UK. The study also engaged in a second-stage Tobit regression analysis, and the findings suggest that population age, population density, and education are influencing factors of efficiency. Coyle et al. [30] argued that although the NHS had engaged in various activities such as critical care and vaccination programs, the measured health output in the UK still experienced a sharp decrease during the period of COVID-19. They concluded that the substantial decline in non-COVID-19 output was attributed to NHS England capacity constraints, and they argued that higher productivity in the uncertain environment for health services can be achieved by increasing capacity in social infrastructure.

In summary, the latest studies covered some issues in terms of the responses of healthcare services to the COVID-19 pandemic, including resource planning, the influence of COVID-19 lockdown, hospital sustainability, and the performance of NHS during the first and second waves of the national lockdown. For these different investigations, a variety of methods have been adopted, including clinical quality indicators, financial indicators, the proposal of the production function of death, and non-parametric data envelopment analysis. Looking at the studies evaluating the performance of NHS in dealing with COVID, no effort has yet been devoted to applying the Multiple Criteria Decision Making (MCDM) models in performance evaluation. We not only fill in the gap in the literature from this perspective by using two MCDM methods to derive performance scores, including COPRAS and TOPSIS, but also filter the unbiased performance scores using LAVRA. Finally, as seen from the literature review, no study has yet attempted to examine what influencing indicators influence the lockdown decision. We contribute to the literature by investigating this topic using the bootstrapped neural network regression, which not only fills in the gap but is also very useful in terms of future policymaking by the government.

5. Stochastic-Fuzzy-Neural MCDM (Multiple-Criteria Decision Making) Approach for Handling Epistemic Uncertainty

5.1. Stage 1: Robust Performance Assessment by Alternative MCDM models

COPRAS (Complex Proportional Assessment) and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) are both decision-making methods widely used in the field of multi-criteria decision analysis. COPRAS, a method developed for complex decision-making scenarios, involves assessing alternatives based on multiple criteria while considering the interdependencies among these criteria. It employs a proportional representation of preferences, allowing decision-makers to capture the intricacies and relationships between different evaluation factors [31]. On the other hand, TOPSIS is a technique designed to determine the best alternative from a set of options by comparing each alternative to an ideal solution and a worst solution. TOPSIS computes the similarity of each alternative to the ideal solution and dissimilarity to the worst solution, ultimately providing a ranking that assists decision-makers in selecting the most favorable option [32]. Both COPRAS and TOPSIS contribute valuable tools to decision analysis, offering systematic and structured approaches for evaluating complex decision problems with multiple criteria. Please see Maredza et al. [10] for details of these two methods.

5.2 Stage 2: Unbiased MCDM Score Decomposition into Latent Vagueness and Randomness Components (LAVRA)

LAVRA approach for determining unbiased MCDM scores departs from the well-established signal processing theory applied to blind image restoration [33]. In this respect, the convolution of two signals is a fundamental operation in signal processing [34]. According to these authors, when image blurring is uniform, it can be modelled as the convolution of a latent original sharp image and its blurring function plus a random noise [35].

Analogously, any given computed MCDM score vector s (i.e., like a blurred image) could be decomposed as the convolution among the bias-free score vector b (i.e., latent or underlying original image), a vagueness component vector v related to criteria measurement (i.e., blurring function), and a random noise vector n (often assumed as half-normally distributed with 0 mean and standard deviation equal to 1), such as:

$$\mathrm{s} = \mathrm{b} \otimes \mathrm{v} \otimes \mathrm{n} \tag{1}$$

where $\otimes$ denotes the binary convolution operator for a discrete set of variables.

With regard to the continuous probability distribution functions, convolution can be apprehended as an operation on two probability distribution functions – a.k.a. pdf´s - (f and g, for instance) that produces a third pdf (f*g). This resulting pdf expresses how the density of one distribution is modified by the other [36]. Convolution is defined as the integral of the product of the two functions after one is reversed and shifted. Some features of the convolution operation are similar to cross-correlation computation, except for the fact that one pdf (either f or g) should be reflected around the vertical axis [37]. Under some circumstances, it is possible to define the convolution of two distributions, where f∗g is a smooth function defined by a pdf given by:

$$\int f(v)g(b-v)dv \quad (2)$$

where $f(v)$ is the pdf for the latent vagueness component V and $g(b)$ is the pdf for the latent unbiased scores B. More generally, it is possible to extend the definition of the convolution uniquely so that the associative law remains valid in the case of compact spaces [38], which turns out to be the case of performance scores and their latent components, which all are defined in the interval [0, 1]:

$$f*(g*h)=(f*g)*h \quad (3)$$

where $h(n)$ is the pdf for the latent random noise component N.

While convolution of probability distributions is a well-known problem in statistics [39], its computational implementation gained momentum after the Fast Fourier Transform (FFT) [40]. Besides, the upcoming parallel computing architectures add to the importance of FFT, in contrast to convolution by direct computations, as proposed by Gupta and Kumar [41]. Nowadays, package distr provides classes for probability distributions within the S4 object-oriented programing concept of R; see Ruckdeschel et al. [39], Ruckdeschel et al. [42]; for further details. Notwithstanding, computational mathematics and its underlying numerical methods are cornerstones for deconvolution. Deconvolution encompasses the numerical procedures taken, observing distributional assumptions and priors, for decomposing a convolutional pdf into its singular pdfs, thus allowing the identification of latent components. In this paper, deconvolution refers to the problem of numerically estimating three latent components: (i) the unbiased MCDM score vector $\mathbf{b}$, (ii) the vagueness or fuzziness component vector v, which is inherent to the epistemic uncertainty that surrounds the computation with different criteria collected from distinct alternatives/data sources, and a noise vector component n, which reflects the intrinsic randomness that may occur in every measurement process. This latent component estimation, or LAVRA, is structured in a two-step approach, as discussed next.

Step 1

In the first step (cf. Model 4), assumptions on pdf´s and their respective prior ranges for B, V, and N random variates are optimized using genetic algorithms, so that the Mean Square Error (MSE) function between the cumulative distribution function – a.k.a. cdf - of the MCDM scores and the cdf of their deconvolution scores is minimal.

$$\begin{aligned}&\text{Minimize: MSE}(\mathrm{c}(\mathrm{s}), \mathrm{c}(\mathrm{b} \otimes \mathrm{v} \otimes \mathrm{n}))\\&\text{Subject to: } \mathrm{l.bound} \leq p \leq \mathrm{u.bound}\end{aligned} \quad (4)$$

where MSE(.) is the Mean-Square Error function between two vectors; c(.) is a cumulative function applied on the original MCDM score and convolution vectors; l.bound and u.bound are, respectively, the lower and upper bound vectors w.r.t. prior assumptions, which are further discussed ahead in Step 1; p is the distributional parameter vector to be optimized, for which the objective function is minimal.

Step 2

Subsequently, in Step 2, a linear linking function for modelling scale and location between both cumulative distributions is computed, yielding a transfer function between original computed MCDM scores and their three latent components. Let k and t be, respectively, the location and the scale parameters of the linear linking function; and let the symbol ^ denote the computed estimates using the optimal p* distributional parameter vector determined in Step 1. Hence, it follows that:

Minimize: $\frac{\sum\left(k + t * c(\hat{s}|p*) - c\left(\widehat{b \otimes v \otimes n}\right)|p*\right)^2}{n}$ Subject to: *k* and *t* free in sign (5)

where n represents the vector size and the objective function is the usual OLS (ordinary least-squares) representation for the MSE function.

Vagueness Component

In the real world, managers frequently deal with imprecise data for taking decisions. In such cases, conventional approaches to address MCDM problems should be no longer used, because conventional modelling ascertains precise or "crisp" values for each alternative under each criterion. While such imprecision or vagueness is often modelled using fuzzy logic, it is worth noting the resemblance between possibilities in fuzzy numbers and probability densities in random "crisp" numbers. This being the case, as regards specifically the case of modelling latent vagueness component, this paper addresses another literature gap, by extending the principles of Two-Dimensional Fuzzy-Monte Carlo (2DFMC) analysis into the underlying vagueness of MCDM models. The 2DFMC approach was firstly proposed by Kentel and Aral [43]. This approach combines the probability and possibility theory; Abdo and Flaus [44] argue that this approach may provide sufficient information for effective decision making. It is also worth noting that this paper is the first piece of research applying 2DFMCA in the stream of convolution of distribution probabilities to explore the impact of data vagueness on the decomposition of their latent components, while building the theoretical gaps between uniform fuzzy numbers and uniform distributions. The underlying idea is to show how one approach can be helpful in gaining insights over the other, that is, randomness helping in apprehending vagueness and vice-versa.

The uniform distribution assumption is the starting point for modelling latent vagueness due to using different data sources (alternatives) for each one of the positive and negative criteria under consideration. Let V~U(a, b), for $0 \leq a < b$, denote the uniform distribution whose density function is f(v) = 1/(b–a) for a < x < b, and f(v) = 0, otherwise. The uniform distribution is typically used as a subjective description of a random variate for which there is only limited or vague data. It is based on a knowledge of the minimum and maximum possible attainable outcomes. For these reasons, the uniform distribution has been called a "lack of knowledge" distribution, thus justifying their choice as a counterpart for the uniform fuzzy number. Now, consider the convolution power operation, or the $\bar{L}$-fold iteration of the convolution of f(v) with itself [45]. Thus, if f(v) is a function on Euclidean space, $\bar{L}$ belongs to a fuzzy number set characterized by membership vector **μ** with size L, the fuzzy-convolution power operation on f(v) is defined by:

$$f(v)^{*\bar{L}} = \frac{[\mu_1 f(v)] * [\mu_2 f(v)] * \ldots * [\mu_L f(v)]}{\sigma(\mu)} \tag{6}$$

where $\sigma(.)$ is the summation function over vector elements and $\mu_1 \ldots \mu_L$ are the elements of the vector **μ**. Readers should note that, while f(v) is a uniform density function for sure, the use of a fuzzy number $\bar{L}$ to compute a novel probability based on the convolution power operation, yielded, per se, a fuzzy probability density function to represent the latent vagueness [46, 47], with parameters a, b, μ, and $\bar{L}$, hence:

$$\overline{\varphi(v|a, b, \mu, \bar{L})} = f(v)^{*\bar{L}} \tag{7}$$

For continuous distributions, which is the case of $\overline{\varphi(v|a, b, \boldsymbol{\mu}, \bar{L})}$, - although other approaches may be available, such as the Fourier transformation of the corresponding characteristic function [48, 49]- the discretization of the respective cdf, based on 0.0001 steps from 0 to 1, may yield enough precision [39] to populate the vector v to be plugged into optimization model (4).

Random Component

The half-normal assumption is adopted as regards the latent noise component, likewise other stochastic MCDM models, such as the stochastic frontiers with performance scores bounded between 0 and 1. The half-normal distribution $HN(m_n, s_n^2)$ is defined by the pdf $h(n|m_n, s_n^2)$ for every n ∈ R+ with mean and variance parameters respectively given by m_n and s_n^2. Analogously to the discussion on the latent vagueness component, the discretization of the respective cdf, $H(n|m_n, s_n^2)$, based on 0.0001 steps from 0 to 1, was the chosen approach to populate the vector n to be plugged into optimization model (4).

Unbiased Scores Component

Lastly, with respect to the latent unbiased score component – that is, free from vagueness and randomness effects -, the beta distribution $B(\alpha, \beta)$ was chosen as the underlying assumption. The beta distribution is a family of continuous probability distributions defined on the interval [0, 1], which pdf $g(b|\alpha, \beta)$ is parameterized by two positive parameters denoted by α and β that control, respectively, the scale and the shape of the distribution. Similarly, discretization of the respective cdf $G(b|\alpha, \beta)$ was also employed to populate the vector b into model (4).

MCDM Scores

While empirical in nature, due to the plethora of alternative existing models and their inherent assumptions (e.g. TOPSIS, VIKOR, COPRAS, MOORA etc), MCDM scores are strongly influenced by the weight vector w assigned to each positive/negative criterion. Hence, it follows that $M(\mathbf{w}|\mathrm{MCDM})$ denotes the empirical distribution of a given MCDM parameterized by the criteria weighting vector. Its kernel pdf can be proxied by $j(s|\mathbf{w})$ and be also discretized to populate the vector s in model (4).

Final Remarks: Latent Component Weights

The objective function of model (4) could also be modified by the addition of a weight vector W for the latent components, such as: MSE(c(s), WTc($\mathbf{b} \otimes \mathbf{v} \otimes \mathbf{n}$)). W elements would reflect a combination of location and scale parameters for each latent component to be decomposed and could be subject to genetic algorithm optimization. Table 2 presents a summary of the distributional assumptions and prior ranges used in the genetic algorithm optimization of model (5).

Table 2
Distributional assumptions and prior ranges for Step 1

Variate	Distribution	Parameters (p)	Domain	Prior lower bounds (l.bound)	Prior upper bounds (u.bound)
Vagueness	Fuzzy convolution power on Uniform distributions	a	R+	0	0.4
		b	R+	0.6	1
		$\boldsymbol{\mu}$	R+	0	1
		L	N+	1	5
Randomness	Half-Normal	m_n	R+	0	1
		s_n	R+	0	1
Unbiased scores	Beta	α	R+	0	4
		β	R+	0	4
MCDM scores	Empirical	w	R+	0	1
Latent component weights	Empirical	W	R	-10	+10

Note: Vector dimensions: dim ($\boldsymbol{\mu}$) = up to 5; dim(**w**) = number of MCDM criteria; dim(**W**) = 3 latent components.

Decision Making: Applications in Management and Engineering
Volume 7, Issue 1 (2024) 473-493

5.3 Stage 3: Neural Network Lockdown Classification

The drivers for lockdown decision is now explored by means of ANNs (Artificial Neural Networks), where classification regressions are computed to discriminate the relative importance of each predictor variable, observing the following functional specification: Lockdown ~ f(Unbiased Scores, Patients Died (lag), MCDM Method, Region), where MCDM Method and Region are dummy variables. In this research, we particularly look at the MLP (Multi-Layer Perceptron) network which has been the most used of ANNs architectures for forecasting [50]. As regards the ANN training, we observed the Connection Weight Approach (CWA) described in Olden et al. [51] and Olden and Jackson [52]. The CWA calculates the product of the raw input-hidden and hidden-output connection weights between each input neuron and output neuron and sums the products across all hidden neurons so that the relative importance of each predictor for taking the lockdown decision is properly mapped. This process was bootstrapped 100 times, yielding the collection of confidence intervals – and the respective statistical significance – for each lockdown predictor.

6. Data Analysis and Discussion of Results

Tables 3 and 4 report the descriptive statistics of the data used in this analysis. Specifically, with respect to Table 3, information is provided for the criteria used: “n” denotes a negative criterion, and “p” denotes a positive criterion. On the other hand, Table 4 provides the descriptive frequencies of the UK regions investigated. During the period of analysis, two national lockdowns were imposed in the UK by Prime Minister Boris Johnson. The first one occurred between 23 March and 4 July, ending with the reopening of pubs and restaurants. The second national lockdown was a 4-week restriction between 5 November and 2 December. A seven-day lagged version of the patients who died, used in LAVRA analysis, was considered as the explained variable of the neural network lockdown classification model.

Looking at the statistics of the standard deviation (SD) in Table 3, we can see that the hospitals used in the sample show a large difference in the average of total beds occupied and maximum total beds occupied, whereas the difference in the sum of patients aged 85+ with COVID-19 and the sum of patients aged 65-84 with COVID-19 is small. The standard deviation provides a measure of absolute dispersion in the original units of the data, while the coefficient of variation provides a standardized measure of relative dispersion, making it easier to compare variability across datasets with different scales or means. A different result is shown when looking at the coefficient of variation. Specifically, there is a large difference in the sum of patients aged 85+ with COVID-19, as well as the sum of patients aged 65-84 with COVID-19, while the level of difference in the average total beds occupied and the maximum total beds occupied is small. Table 4 illustrates the distribution of hospitals across different regions in the analysis. The Midlands emerges as the region with the highest representation, accounting for 18.18% of the total hospitals. Following closely is London, representing 16.58%, while the Northeast and Yorkshire together contribute 15.51%. The Northwest and Southeast regions display notable shares at 14.97% and 13.9%, respectively. Meanwhile, the East of England constitutes 11.23% of the hospitals, and the Southwest region comprises 9.63%.

Figure 1 displays the density plots of COPRAS and TOPSIS scores, revealing a noteworthy distinction between the two sets. It is intriguing to observe that both sets exhibit effective discrimination. Notably, the performance scores are higher when considering the "distance to ideal solutions" assumption compared to the "utility function" assumption. This observation implies that, from a managerial standpoint, having higher criteria slacks could potentially be beneficial in achieving elevated healthcare performance. Our results, to a certain extent, challenge the arguments from Coyle et al. [30], which advocated that the UK NHS suffered from low performance during the COVID-

19 pandemic from the perspective of reduced outputs. It is indicated from our results that the performance of the UK NHS during the COVID-19 period was low, as reflected by the performance scores estimated from COPRAS, but this is not really the case for the performance scores estimated from TOPSIS. Therefore, we can deduce from this finding that the level of performance of the UK NHS during the COVID-19 pandemic will depend on the data used and the estimation method in the data analysis.

Tables 5 and 6 report the results for LAVRA decomposition in steps 1 and 2, respectively, performed simultaneously for both sets of TOPSIS and COPRAS scores. These results indicate that these MCDM scores could not only be successfully decomposed into three major components (cf. Table 5) – unbiased, randomness, and vagueness – but could also be associated with a quasi-perfect linear transfer function to LAVRA model assumptions (cf. Table 6 and Figure 2) based on their cumulative sum distribution, suggesting that the score rank-order was preserved during the decomposition process. It is interesting to note that, based on Figure 3, the relative importance of each decomposed component in each MCDM can vary. While the vagueness component is more prominent in the TOPSIS model, the random component is more relevant in the COPRAS model. This may be explained by the fact that the underlying epistemic uncertainty captured by distinct MCDM is different. While “ideal solutions” may be more subjected to vagueness than to randomness – After all, what are the actual ideal limits for the positive and negative criteria? – “utility functions” seem to be more impacted by random variations in collected data than by fuzziness with respect to its practical meaning for decision-making. That is, it is clearer to identify which criteria are more useful to increase performance levels than to conjecture on their ideal boundaries.

Figure 4 reports on the bootstrapped neural network results for classification: 1 indicates lockdown, 0 otherwise. Results suggest that the lockdown decision is triggered by a combination of predictive factors, such as (i) geographic region – where Northeast/Yorkshire, Southwest, London, Northwest, and Southeast appear in decreasing order of importance; (ii) the death toll, which positively impacted the lockdown decision with a lag of seven days; (iii) the unbiased performance scores, which negatively impacted the lockdown decision. In other words, the lockdown decision appears to be taken based on an undoubted performance drop of the healthcare system (free from vagueness and random components), associated with a relatively recent rise in the death toll while still subjected to the specifics of each region. We think the results are very reasonable and give a full picture of the lockdown drivers in a very comprehensive way. We further think that these three different factors have a certain level of correlation. For example, the priority of lockdown in a specific geographic region will be based on the degree of seriousness of COVID-19 in terms of the mortality rate as well as the number of positive cases. On the other hand, the mortality rate and the number of positive cases would have a significant impact on the NHS hospitals' capacity to accommodate COVID patients and further have an influence on the level of hospital performance.

Our results are in contrast with Kuosmanen et al. [24], who report that the hospitals in London were the best performers, whereas the ones in North East and North West were the worst. This is mainly attributed to the fact that different methods were adopted by the studies. Our findings are also different from the ones of Ferraresi et al. [53] from the perspective that the latter focuses on the investigation into the lockdown decisions across different countries, while we focus on the examination of the determinants of lockdown decisions within a specific country. Compared to Ferraresi et al. [53], we think our research could be very useful and effective in providing specific and concrete policy implications within the country at the micro level to deal with the COVID-19 pandemic.

Table 3
Descriptive statistics for criteria used in TOPSIS and COPRAS

Variable	Type	Min	Max	Median	Mean	SD	CV	Skewness	Kutosis
Total beds-occupied (average)	n	0.00	2315.43	407.14	465.13	324.61	0.70	1.50	4.08
Total beds-occupied (max)	n	0.00	3347.00	435.00	494.10	341.99	0.69	1.58	4.84
Total bed occupied by COVID-19 (average)	n	0.00	689.43	9.14	32.29	56.29	1.74	3.59	18.68
Total bed occupied by COVID-19 (max)	n	0.00	791.00	12.00	37.36	62.91	1.68	3.58	19.41
Mechanical ventilation beds (average)	n	0.00	164.43	7.86	13.99	20.31	1.45	2.68	9.07
Mechanical ventilation beds (max)	n	0.00	177.00	10.00	16.37	22.60	1.38	2.54	8.17
Mechanical ventilation beds by COVID-19 (average)	n	0.00	141.57	0.14	3.79	9.24	2.44	6.41	61.08
Mechanical ventilation beds by COVID-19 (max)	n	0.00	150.00	1.00	4.68	10.71	2.29	6.02	53.91
COVID 19 discharges (sum)	p	0.00	520.00	4.00	16.80	30.74	1.83	3.97	28.68
Patients with COVID-19 aged 65-84 (sum)	n	0.00	163.00	0.00	2.34	5.96	2.55	8.56	143.05
Patients with COVID-19 (sum)	n	0.00	441.00	1.00	5.64	14.20	2.52	10.08	203.28
Patients with COVID-19 aged 85+ (sum)	n	0.00	68.00	0.00	1.10	2.84	2.59	8.04	123.54
Inpatients with COVID-19 (sum)	p	0.00	574.00	3.00	17.82	37.49	2.10	4.70	36.29
Inpatients with COVID-19 aged 65-84 (sum)	p	0.00	234.00	1.00	7.81	16.64	2.13	4.27	27.43
Inpatients with COVID-19 aged 85+ (sum)	p	0.00	112.00	0.00	2.88	6.69	2.32	5.23	43.96
Staff absent - sickness and self-isolation (average)	n	0.00	4047.00	326.57	418.35	360.72	0.86	2.68	12.25
Staff absent - sickness and self-isolation (max)	n	0.00	11926.00	355.00	458.11	433.71	0.95	6.70	126.78
Patients died (sum)	n	0.00	218.00	1.00	5.51	12.58	2.28	4.92	40.10

Notes: SD represents standard deviation and CV stands for coefficient of variation. n/p represents negative/positive criterion.

Table 4
Frequency distribution

Region	Proportion (%)
East of England	11.23
London	16.58
Midlands	18.18
Northeast and Yorkshire	15.51
Northwest	14.97
Southeast	13.9
Southwest	9.63

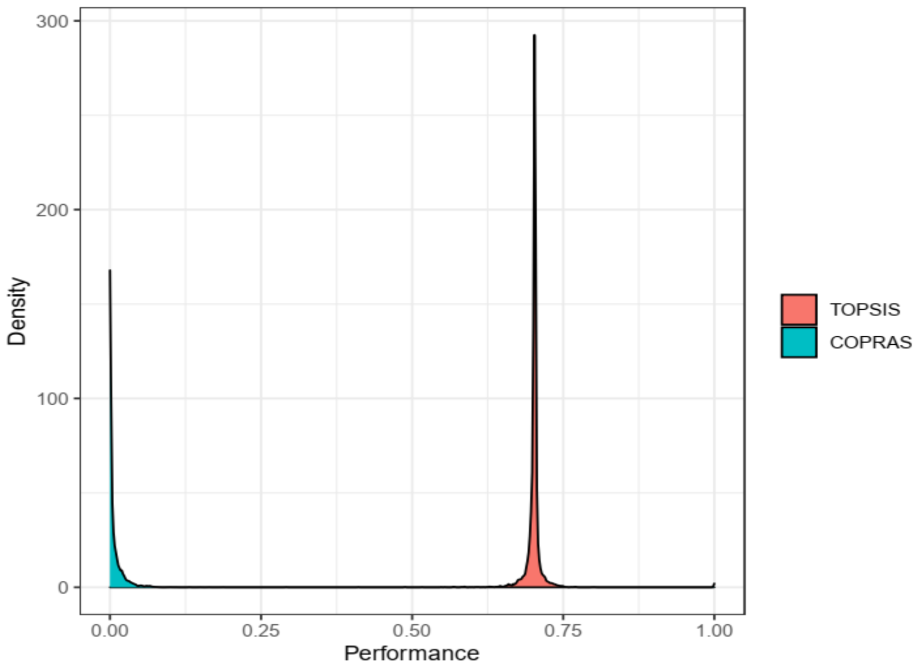

Fig. 1. Density plot comparison between TOPSIS and COPRAS performance scores.

Table 5.
LAVRA Optimal parameters obtained in Step 1

Optimal Parameters (p*)	Solution
a	0.7727777
b	0.2119855
μ	0.4572753 0.3277117 0.369686 0.371979 0.4675602
L	5
m_n	0.4773778
s_n	0.5108831
α	2.055437
β	2.838213
w	0.5818023 0.4082563 0.2191872 0.6882252 0.6208299 0.4106004 0.5031035 0.4678186 0.3981063 0.5220427 0.3866621 0.5714014 (twelve criteria)
W	0.2665797 (v) -4.075936 (b) 0.2886252 (n)

Minimal MSE= 1719.997

Table 6
Location and scale parameters obtained in Step 2

Optimal Parameters	Solution
k	1.350e-01
t	2.771e-01

Residual standard error: 1.431 on 4972 degrees of freedom
Multiple R-squared: 0.9999, Adjusted R-squared: 0.9999
F-statistic: 7.571e+07 on 1 and 4972 DF, p-value: < 2.2e-16

Decision Making: Applications in Management and Engineering
Volume 7, Issue 1 (2024) 473-493

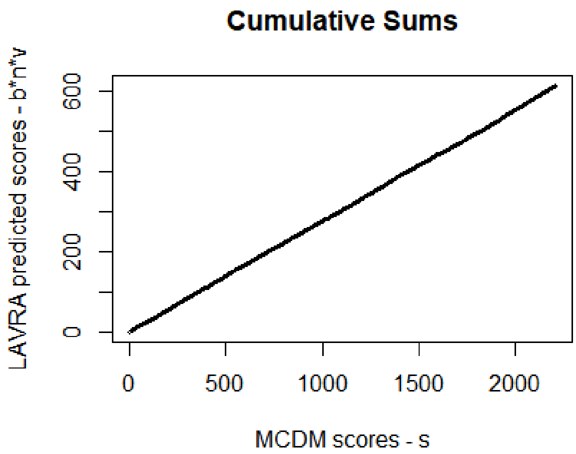

Fig.2. Cumulative sums for model (5) objective function based on ***p****

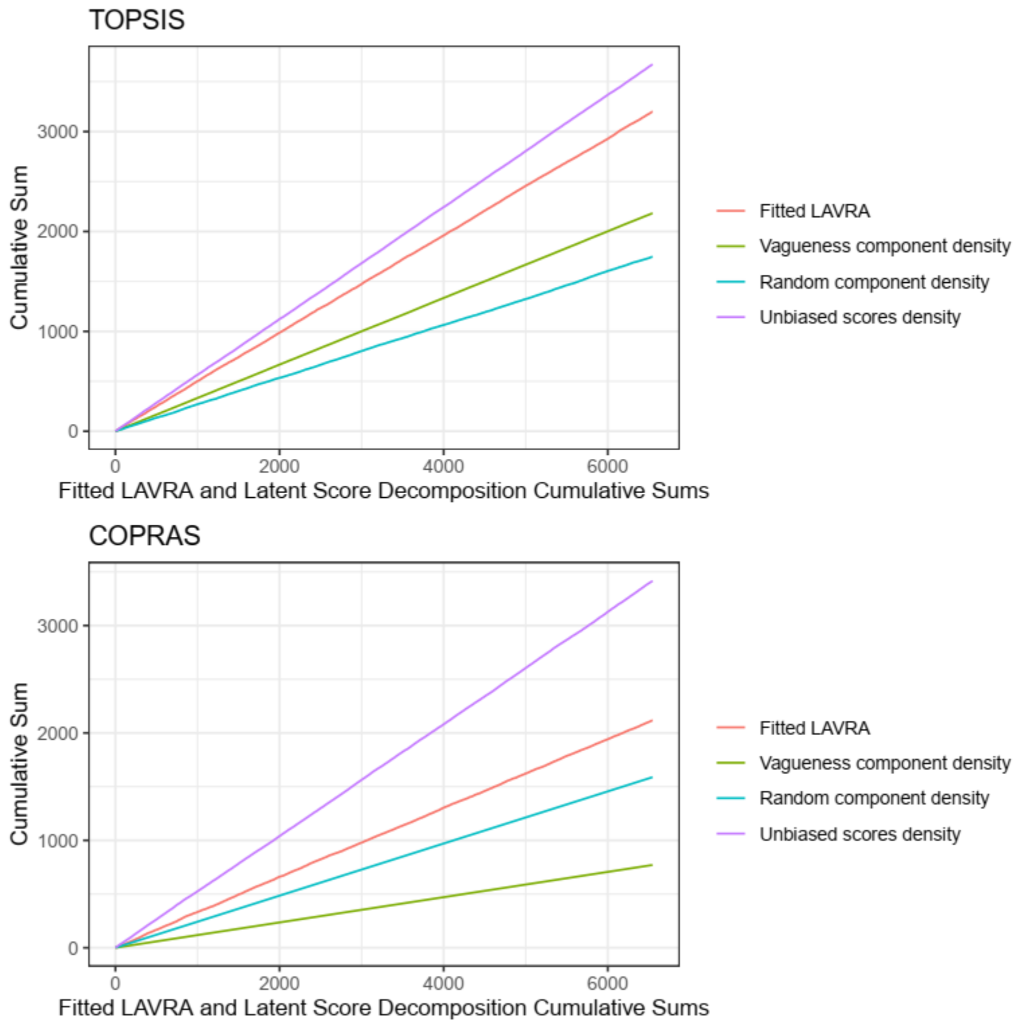

Fig. 3. LAVRA cumulative sum decomposition for each MCDM model

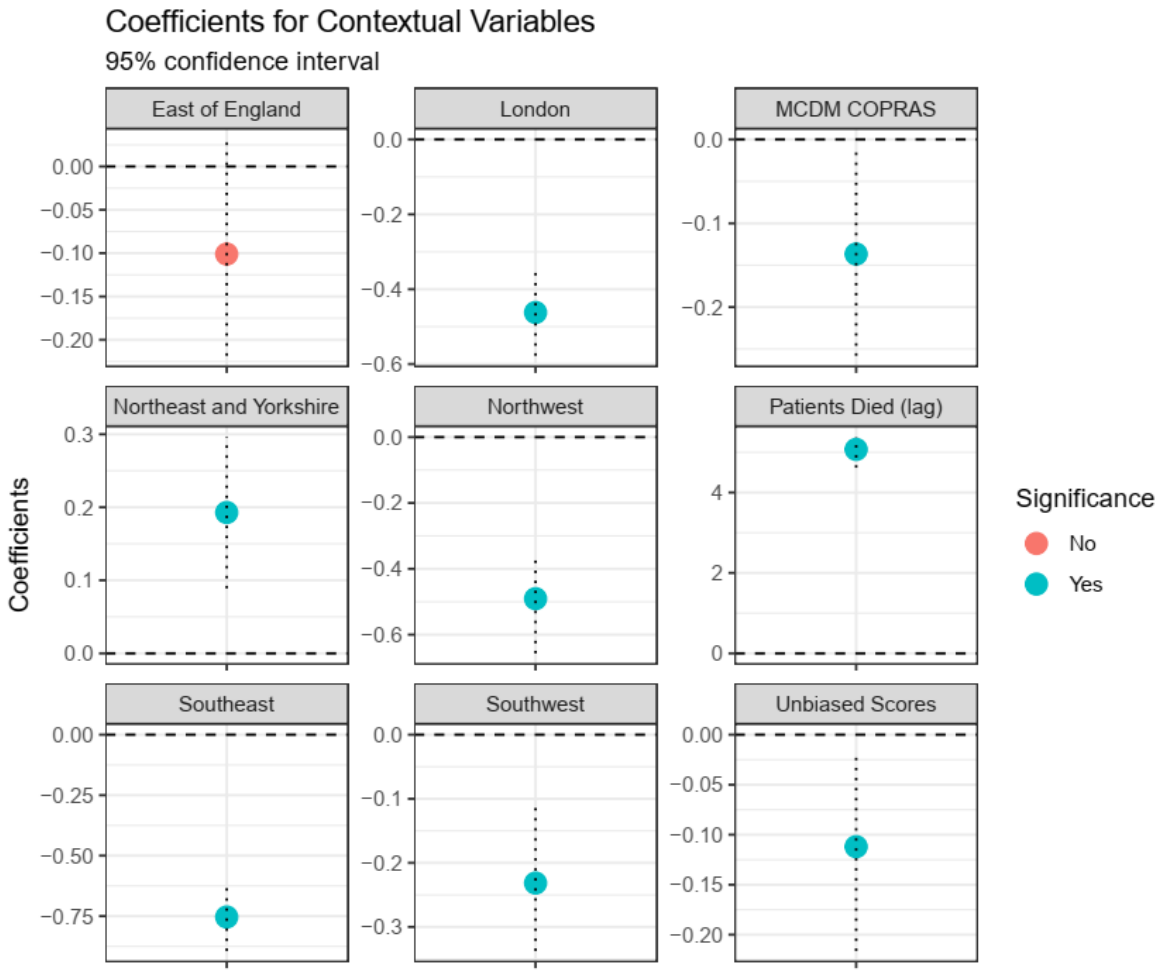

Fig. 4. Bootstrapped results for Olden´s sensitivity analysis in terms of signs and relative importance. (MCDM TOPSIS and Midlands as reference)

7. Conclusions

The outbreak of COVID-19 since the start of 2020 has affected more than 200 countries around the world. The characteristics of the fast and easy spread of the virus, as well as the disastrous effects on people's lives, have significantly affected the level of economic activities. Around the world, all countries experienced a degree of economic recession except China, which still had a level of economic growth. COVID-19 has not only been affecting the level of economic activity, but also relevant policies made by the government, such as social contact rules and stay-at-home rules, have substantially reduced the level of social activities that can be engaged in by the people. The purpose of these measures was not only to slow down the spread of the virus but also to protect the national health service system, enabling it to have enough capability to cope with COVID patients. The national health service system has been playing a key role during the pandemic by slowing down the spread of the virus through consistently admitting COVID patients and engaging in relevant treatments. However, there have been several challenges that prevented them from performing even better. The challenges are mainly related to the limited amount of resources in terms of treatment or protection-related materials and labor; substantial unexpected increases in the number of COVID patients, which exceeded the hospitals' capacity; as well as the increased burden from non-COVID patients.

The current study distinguishes itself from other studies by being the pioneer to evaluate the performance of NHS during the pandemic and the lockdown drivers. More specifically, the research was undertaken through three steps. In the first stage, performance measurement is alternatively addressed either by computing partial utility functions of each performance criterion using COPRAS (Complex Proportional Assessment) or by measuring the partial distances of each criterion to its respective ideal solution using TOPSIS (Technique for Order Preference by Similarity to Ideal Solution). In the second stage, the novel LAVRA approach is used to filter unbiased performance scores apart from vague and random components. In the third stage, a bootstrapped neural network regression is proposed to classify the lockdown drivers in terms of performance, deaths, and geographic regions.

The findings suggest that using the TOPSIS analysis, the performance scores range from 0.65 to 0.75; however, when we analyze the performance scores using the COPRAS, the performance score was significantly decreased. No matter which MCDM method is used, the findings suggest that there can be a further improvement in the operation and management of the NHS hospitals in dealing with the pandemic. In terms of the lockdown drivers, our results show that the combination of three different considerations (the geographical regions, the death toll, as well as the unbiased hospital performance score) will affect the decisions. We argue that these three different factors are correlated with each other; therefore, the priority for the NHS and government in dealing with the pandemic in the future is to take relevant measures to slow down the spread of the virus and also focus on the capacity expansion of hospitals in dealing with the unexpected increase in the number of patients during the pandemic.

More specifically, we have the following policy implications: 1) Northeast and Yorkshire should be given priority in making the lockdown decisions. This was evidenced by the news from BBC revealing that there was more death and lower wages in the Northeast compared to other parts of the country, while in terms of Yorkshire, the case is slightly different from the one in the Northeast. BBC reports that the type of jobs in Yorkshire was one factor leading to the high infection rate in Yorkshire. Deprivation, insecure employment, as well as living in multi-occupancy homes are additional factors resulting in the higher infection rate. Therefore, it seems that economic development should be focused on in the Yorkshire area, which will not only boost prosperity and increase employment but also is supposed to reduce the infection rate for any COVID-like health crisis in the future; 2) We saw that the UK government reported the number of positive cases as well as the number of death on a daily basis; it seems that lots of positive cases indicate a growing speed in the spread of COVID-19. However, our research shows that to some extent, the number of deaths is also very important, especially for the government to make lockdown decisions; 3) Further emphasis should be given to improving the performance of NHS hospitals from the perspective of increasing the capacity of admitting COVID patients; relevant mechanisms should be considered not only to expand the capacity but also to improve the ability of NHS hospitals to deal with emergency issues. This, to some extent, can be facilitated by the improvement in health-related technologies.

Future studies can further update the data to the most recent period to see whether COVID vaccinations have any influence on the improvement in the performance of NHS hospitals. In addition, our proposed innovative method can be used to analyze the performance of hospitals in other countries, and relevant comparisons with the one of the UK can be made. Finally, the combination of different MCDM approaches, such as the COPRAS and TOPSIS, may result in theoretical incompatibilities regarding the definition of utility functions or the definition of criteria weights. Moreover, the combination of all the employed techniques throughout the three stages of the proposed approach is unnecessarily complex. Future studies can apply the non-parametric data envelopment analysis for the performance evaluation and compare the results with the ones from the current study.

Author Contributions

Conceptualization, P.W., J. A., Y.T. and S.A.E.; methodology, P.W. and J. A.; software, J.A.; formal analysis, P.W. J. A., Y. T. and S.A.E.; data curation, P. W., J. A. and Y.T.; writing—original draft preparation, P. W., J. A. and Y.T.; writing—review and editing, S.A.E.; visualization, J. A.; supervision, P.W.; project administration, Y.T.; All authors have read and agreed to the published version of the manuscript.

Funding

This research received no external funding.

Data Availability Statement

The data used in this paper will be available upon request from the corresponding author.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This research was not funded by any grant.

References

[1] Verikios, G. (2020). The dynamic effects of infectious disease outbreaks: The case of pandemic influenza and human coronavirus. Socio-Economic Planning Sciences, 71, 100898. https://doi.org/10.1016/j.seps.2020.100898

[2] Maity, S., & Barlaskar, U.R. (2022). Women's political leadership and efficiency in reducing COVID-19 death rate: An application of technical inefficiency effects model across Indian states. Socio-Econ Planning Sciences, 82(B), 101263. https://doi.org/10.1016/j.seps.2022.101263

[3] Worldometer (2022). COVID-19 Coronavirus Pandemic. https://www.worldometers.info/coronavirus/. Accessed 8th April 2022.

[4] Manupati, V.K., Ramkumar, M., Baba, V., & Agarwal, A. (2021). Selection of the best healthcare waste disposal techniques during and post COVID-19 pandemic era. Journal of Cleaner Production, 281, 125175. https://doi.org/10.1016/j.jclepro.2020.125175.

[5] Lin, C-K., Chen, J.K.C., & Ho, H-H. (2021). BIM for Smart Hospital Management during COVID-19 Using MCDM. Sustainability, 13, 6181. https://doi.org/10.3390/su13116181

[6] Mosadeghi, R., Warnken, J., Tomlinson, R., & Mirfenderesk, H. (2013). Uncertainty analysis in the application of multi-criteria decision-making methods in Australian strategic environmental decisions. Journal of Environmental Planning and Management, 56(8), 1097-1124. https://doi.org/10.1080/09640568.2012.717886

[7] Ferreira, D., Marques, R., & Nunes, A. (2021). Pay for performance in health care: a new best practice tariff-based tool using a log-linear piecewise frontier function and a dual–primal approach for unique solutions. Operational Research, 21, 2101-2146. https://doi.org/10.1007/s12351-019-00502-3

[8] Rocha, A, Costa, A.S., Ferreira, D.C., Figueira J.R., & Marques, R.C. (2021). Quality assessment of the Portuguese public hospitals: A multiple criteria approach. Omega, 105, 102505. https://doi.org/10.1016/j.omega.2021.102505

[9] Roy, B. (1968). Classement et choix en présence de points de vue multiples (la méthode ELECTRE). RIRO, 2(8). 57-75. https://doi.org/10.1051/ro/196802V100571

[10] Maredza, A., Wanke, P., Antunes, J., Pimenta, R., & Tan, Y. (2022). Social welfare and bank performance: evidence from a stochastic neural hybrid MCDM approach. Journal of Economic Studies, 49(7), 1137-1158. https://doi.org/10.1108/JES-05-2021-0236

[11] Antunes, J., Hadi-Vencheh, A., Jamshidi, A., Tan, Y., & Wanke, P. (2023). TEA-IS: A hybrid DEA-TOPSIS approach for assessing performance and synergy in Chinese health care. Decision Support Systems, 171, 113916. https://doi.org/10.1016/j.dss.2022.113916
[12] Simar, L., & Wilson, P.W. (2007). Estimation and inference in two-stage, semi-parametric models of production processes. Journal of Econometrics, 136(1), 31-64. https://doi.org/10.1016/j.jeconom.2005.07.009
[13] Rafighi, E., Poduval, S., Legido-Quigley, H., & Howard, N. (2016). National Health Service Principles as Experienced by Vulnerable London Migrants in "Austerity Britain": A Qualitative Study of Rights, Entitlements, and Civil-Society Advocacy. International Journal of Health Policy and Management, 5(10), 589-597. https://doi.org/10.15171/ijhpm.2016.50
[14] Oliver, A. (2005). The English National Health Service: 1979-2005. Health Economics, 14(S1), S75-S99. https://doi.org/10.1002/hec.1029
[15] Talbot-Smith, A., & Pollock, A.M. (2006). The New NHS. Routledge, London.
[16] Pettigrew, L.M., Kumpunen, S., Rosen, R., Posaner, R., & Mays, N. (2019). Lessons for 'large-scale' general practice provider organisations in England from other inter-organisational healthcare collaborations. Health Policy, 123(1), 51-61. https://doi.org/10.1016/j.healthpol.2018.10.017
[17] Checkland, K., Coleman, A., McDermott, I., Segar, J., Miller, R., Petsoulas, C., Wallace, A., Harrison, S., & Peckham,S. (2013). Primary care-led commissioning: applying lessons from the past to the early development of clinical commissioning groups in England. British Journal of General Practice, 63(614), e611-e619. https://doi.org/10.3399/bjgp13X671597
[18] Hyde, P., Granter, E., Hassard, J., & McCann, L. (2016). Deconstructing the welfare state. Routledge, London.
[19] NHS digital (2022). NHS Workforce Statistics - September https://digital.nhs.uk/data-and-information/publications/statistical/nhs-workforce-statistics/september-2021 (accessed 4th December 2023)
[20] Kings Funds (2022). The NHS budget and how it has changed. https://www.kingsfund.org.uk/projects/nhs-in-a nutshell/nhs-budget (accessed 4th December 2023)
[21] Full Fact (2019) Spending on the NHS in England. https://fullfact.org/health/spending-english-nhs/. Accessed 4th December 2023.
[22] NHS Improvement (2021). Consolidated NHS provider accounts 2019/20. https://www.england.nhs.uk/wp-content/uploads/2021/01/consolidated-nhs-provider-accounts-19-20.pdf. (accessed 4th December 2023)
[23] Oomman, S., & Todd, E. (2021). Impact of COVID-19 lockdown on A&E performances in an NHS Foundation Trust. Postgraduate Medical Journal, J 97(1143), 48-50. https://doi.org/10.1136/postgradmedj-2020-138894
[24] Rathnayake, D., Clarke, M., & Jayasinghe, V.I. (2021). Health system performance and health system preparedness for the post-pandemic impact of COVID-19: A review. International Journal of Healthcare Management, 14(1), 250-254. https://doi.org/10.1080/20479700.2020.1836732
[25] Kruse, F.M., & Jeurissen, P.P.T. (2020). For-Profit Hospitals Out of Business? Financial Sustainability During the COVID-19 Epidemic Emergency Response. International Journal of Health Policy and Managmeent, 9(10), 423-428. https://doi.org/10.34172/ijhpm.2020.67
[26] Kuosmanen, T., Tan, Y., & Dai, S. (2021). Performance analysis of English hospitals during the first and second waves of the coronavirus pandemic. Health Care Management Science, 26,447-460. https://doi.org/10.1007/s10729-023-09634-7
[27] Hamzah, N., Yu, M-M., & See, K.F. (2021). Assessing the efficiency of Malaysia health system in COVID-19 prevention and treatment response. Health Care Management Science, 24, 273-285. https://doi.org/10.1007/s10729-020-09539-9
[28] Breitenbach, M.C., Ngobeni, V., & Aye, G.C. (2021). Global Healthcare Resource Efficiency in the Management of COVID-19 Death and Infection Prevalence Rates. Frontiers in Public Health, 9, 638481. https://doi.org/10.3389/fpubh.2021.638481
[29] Lupu, D., & Tiganasu, R. (2022). COVID-19 and the efficiency of health systems in Europe. Health Economic Review, 12: 14. https://doi.org/10.1186/s13561-022-00358-y
[30] Coyle, D., Dreesbeimdieck, K., & Manley, A. (2021). Productivity in UK healthcare during and after the COVID-19 Pandemic. National Institute Economic Review, 258, 90-116. http://doi.org/10.1017/nie.2021.25
[31] Antunes, J., Tan, Y., Wanke, P., & Jabbour, C. J. C. (2023). Impact of R&D and innovation in Chinese road transportation sustainability performance: A novel trigonometric envelopment analysis for ideal solutions (TEA-IS). Socio-Economic Planning Sciences, 87(A), 101544. https://doi.org/10.1016/j.seps.2023.101544
[32] Wanke, P., Antunes, M. A. A., Filgueira, A. L. L., Michelotto, F., Tardin, I. G. E., & Tan, Y. (2023). Efficiency analysis of the OECD countries' long-term productivity using DEA and TOPSIS approaches Benchmarking, https://doi.org/10.1108/BIJ-11-2021-0662

[33] Teranishi, R., Goto, T., & Nagata, T. (2019). Improvement of Robustness in Blind Image Restoration Method Using Failing Detection Process. Journal of Image and Graphics, 8(3), 1–8. 85-92.

[34] Nah, S., Kim, T.H., & Lee, K.M. (2017). Deep Multi-scale Convolutional Neural Network for Dynamic Scene Deblurring. IEEE International Conference on Computer Vision and Pattern Recognition (CVPR), 257-265. https://doi.ieeecomputersociety.org/10.1109/CVPR.2017.35

[35] Damelin, S., & Miller, W. (2011). The Mathematics of Signal Processing. Cambridge University Press, Cambridge.

[36] Dominguez-Torres, A. (2010). The origin and history of convolution I: continuous and discrete convolution operations. https://www.slideshare.net/Alexdfar/origin-adn-history-of-convolution. (accessed 5th December 2023)

[37] Sobolev, V.I. (2001). Convolution of functions. https://encyclopediaofmath.org/wiki/Convolution_of_functions. (accessed 5th December 2023)

[38] Hörmander, L. (1983). The analysis of linear partial differential operators I. Springer, Heidelberg.

[39] Ruckdeschel, P., Kohl, M., Stabla, T., & Camphausen, F. (2014). distr: S4 Classes for Distributions -A Manual for Packages distr, distrEx, distrEllipse, distrMod, distrSim, distrTEst, and distrTeach. Version 2.5. http://CRAN.R-project.org/package=distrDoc. (accessed 5th December 2023)

[40] Cooley, J.W., & Tukey, J.W. (1965). An Algorithm for the Machine Calculation of Complex Fourier Series. Mathematics of Computation, 19(90), 297-301. https://doi.org/10.2307/2003354

[41] Gupta, A., & Kumar, V. (1993). The scalability of FFT on parallel computers. IEEE Transactions on Parallel and Distributed Systems, 4 (8), 922-932. https://doi.org/10.1109/71.238626

[42] Ruckdeschel, P., & Kohl, M. (2014). General purpose convolution algorithm in {S}4 classes by means of FFT. Journal of Statistical Software, 59(4), 1-25. http://dx.doi.org/10.18637/jss.v059.i04

[43] Kentel, E., & Aral, M.M. (2005). 2D Monte Carlo versus 2D Fuzzy Monte Carlo health risk assessment. Stochastic Environmental Research and Risk Assessment, 19, 86-96. https://doi.org/10.1007/s00477-004-0209-1

[44] Abdo, H., & Flaus, J.M. (2016). Uncertainty quantification in dynamic system risk assessment: a new approach with randomness and fuzzy theory. International Journal of Production Research, 54(19), 5862-5885. https://doi.org/10.1080/00207543.2016.1184348

[45] Chrouda, M.B., Oued, M.E., & Ouerdiane, H. (2002). Convolution calculus and applications to stochastic differential equations. Soochow Journal of Mathematics, 28(4), 375–388.

[46] Buckley, J.J. (2004). Fuzzy Probabilities and Fuzzy Sets for Web Planning. Springer, Heidelberg.

[47] Buckley, J.J. (2005). Simulating Fuzzy Systems. Springer, Heidelberg.

[48] Chung, K.L. (1974). A Course in Probability Theory. 2nd edition. Academic Press, New York.

[49] Lumely, T. (2007). Tail Area of Sum of Chi-square Variables. https://stat.ethz.ch/pipermail/r-help/2007-March/128685.html. (accessed 5th December 2023)

[50] Mubiru, J., & Banda, E.J.K.B. (2008). Estimation of monthly average daily global solar irradiation using artificial neural networks. Solar Energy, 82(2), 181-187.

[51] Olden, J.D., Joy, M.K., & Death, R.G. (2004). An accurate comparison of methods for quantifying variable importance in artificial neural networks using simulated data. Ecological Modelling, 178(3-4), 389-397. https://doi.org/10.1016/j.ecolmodel.2004.03.013

[52] Olden, J.D., & Jackson, D.A. (2002). Illuminating the "black-box": a ramdomization approach for understanding variable contributions in aritificial neural networks. Ecological Modelling, 154(1-2), 135-150. https://doi.org/10.1016/S0304-3800(02)00064-9

[53] Ferraresi, M., Kotsogiannis, C., Rizzo, L., & Secomandi, R. (2020). The 'Great Lockdown' and its determinants. Economics Letters, 197, 109628. https://doi.org/10.1016/j.econlet.2020.109628