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# GENERALIZED Z-FUZZY SOFT $\beta$-COVERING BASED ROUGH MATRICES AND ITS APPLICATION TO MAGDM PROBLEM BASED ON AHP METHOD

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***Abstract:*** *Fuzzy sets, rough sets and soft sets are different mathematical tools mainly developed to deal with uncertainty. The combination of these theories has a wide range of applications in decision analysis. In this paper, we defined a generalized Z-fuzzy soft $\beta$-covering-based rough matrices. Some algebraic properties are explored for this newly constructed matrix. The main aim of this paper is to propose a novel MAGDM model using generalized Z-fuzzy soft $\beta$-covering-based rough matrices. A MAGDM algorithm based on AHP method is created to recruit the best candidate for an assistant professor job in an institute and a numerical example is presented to demonstrate the created method.*

***Keywords:*** *$\beta$-level soft set, Fuzzy soft $\beta$-adhesion, Generalized Z-fuzzy soft $\beta$-covering based rough matrix, AHP.*

## 1. Introduction

Different theories have been proposed to deal with uncertainty in data mining when conventional mathematics fails. To handle problems containing uncertainty, many theories, such as probability, fuzzy sets, soft sets, rough sets, and a combination of these theories have been utilized. Each of these concepts appears to have its own set of constraints and limitations. Zadeh (1965) introduced fuzzy sets to handle uncertainties in datasets. Pawlak (1982) proposed the concept of a rough set (RS). Rough set is used in data analysis to discover basic patterns in data, eliminate redundancies, and develop decision rules. Sharma et al. (2018, 2021, 2022) developed various hybrid methods using rough set theory to create decision rules useful for real-world problems. Several scholars are working on this concept, which is being applied to a variety of fields such as in (Greco et al., 2001). Covering rough

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sets (CRS) is an important study topic as an extension of rough sets. CRS is a useful tool that allows academics to examine uncertainty in a broader context. Because of its broad variety of applications, CRS has attracted a large number of researchers. Zhu and Wang (2007, 2012) proposed a variety of CRS models. Dubois and Prade (1990) have proposed two novel models such as fuzzy rough sets (FRS) and rough fuzzy sets (RFS).

Molodtsov (1999) proposed a soft set theory to describe a useful mathematical tool to deal with multi-attribute uncertainty. Soft set theory describes a wide range of information and computational activities. Maji et al. (2002, 2003) developed a number of operations on soft sets. Bipolar soft sets are capable of dealing with ambiguity. Tufail et al. (2022) combined bipolar soft set with rough set and proposed a novel MCGDM algorithm. Ali (2011) examined the relationships between rough sets, fuzzy sets and soft sets. Cagman and Enginoglu (2010, 2012) presented soft matrix and fuzzy soft matrix and explored their algebraic properties. Vijayabalaji (2014) introduced the generalized matrix representation for soft rough matrices. Muthukumar and Krishnan (2018) presented a novel way of representing the fuzzy soft rough matrix in a generalized manner. Yüksel et al. (2014) proposed soft covering-based rough sets (SCRS) and developed an application that combined rough sets and covering soft sets. Zhan and Wang (2019) defined five forms of SCRS and described the relationship between SRS and SCRS. Soft rough fuzzy-covering (SRFC) and soft fuzzy rough covering (SFRC) models are presented by Zhan and Sun (2019). Zhang and Zhan (2019) contributed fuzzy soft $\beta$-coverings and defined four different fuzzy soft $\beta$-covering based fuzzy rough sets (FS$\beta$CFRS). Yang (2022) proposed fuzzy covering based rough set over dual universe as an extension of single universe concept.

A multiple attributes group decision-making (MAGDM) problem is one in which a group of qualified specialists examine and select the best alternative from a collection of objects based on their attributes. MAGDM is used in variety of fields. Research in MAGDM problems using fuzzy sets, rough sets and soft sets has increased in recent years (Gurmani et al., 2022). Due to the ambiguity in choice objects and decision-makers desire, it is not easy to explicit the decision maker's idea in precise value. To efficiently deal with vagueness information, a generalized Z-fuzzy soft $\beta$-covering based rough matrix is introduced into the MAGDM problem. Saaty (1980) introduced Analytical Hierarchy Process (AHP), which is a well-known MCDM technique. In MCDM issues, AHP establishes a hierarchy of components and determines values for each of those elements via pairwise comparisons. AHP is a technique for guiding decision-making processes. The AHP framework assists firms in making various decisions by assessing and evaluating criteria. By following the phases of our proposed technique, we can make better-informed decisions.

### 1.1. Motivation

Vijayabalaji (2014) first introduced the idea of converting soft rough sets into soft rough matrices and they generalised the concept of soft rough matrices by redefining it. Muthukumar and Krishnan (2018) proposed the concept of generalized fuzzy soft rough matrices and developed a novel decision-making model using this technique. Inspired by these concepts, the idea of Generalized Z-fuzzy soft $\beta$-covering based rough (Z-GFS$\beta$CR) matrices is presented and applied to decision-making problems using a $\beta$-level soft set. The purpose of this manuscript is to establish a foundation for Z-GFS$\beta$CR matrices to handle uncertainty problems. We first proposed the theoretical concepts of Z-GFS$\beta$CR matrices and their operations, which are more important for conducting theoretical studies in the extension of fuzzy soft rough set theory.

The summary of the present paper is systematized as follows. In section 2, all the preliminaries needed to understand the following sections are provided. In section 3, we define a new type of Z-fuzzy soft $\beta$-covering based rough set (Z-FS$\beta$CRS) using fuzzy soft $\beta$-adhesion formed from $\beta$-level soft set. We develop a matrix representation for Z-FS$\beta$CRS. Further, we define a generalized matrix representation for Z-FS$\beta$CRS known as generalized Z-fuzzy soft $\beta$-covering based rough (Z-GFS$\beta$CR) matrix and we examine some algebraic properties of Z-GFS$\beta$CR matrices. In section 4, we create a decision making algorithm using Z-GFS$\beta$CR matrices. In section 5, the conclusion is addressed.

## 2. Preliminaries

In this section, the preliminaries that are necessary to understand the below sections are described. Let $\Omega$ be a finite universal set and $B \subseteq E$ be the attribute set throughout this paper.

*Definition 1* (Molodtsov, 1999). A pair $K = (\widetilde{N}, B)$ be a soft set over $\Omega$, if $\widetilde{N}$ is a mapping defined by $\widetilde{N}: B \to P(\Omega)$ where $P(\Omega)$ denote the power set of $\Omega$.

*Definition 2* (Feng et al., 2010). A soft set $(\widetilde{N}, B)$ over $\Omega$ is known as a full soft set, if $\cup_{b_j \in B} \widetilde{N}(b_j) = \Omega$.

*Definition 3* (Yüksel et al., 2014). A full soft set $(\widetilde{N}, B)$ is known as a soft covering (SC), if $\widetilde{N}(b_j) \neq \emptyset$, for each $b_j \in B$, and it is denoted by $\tilde{C}_K$. Let $(\Omega, \tilde{C}_K)$ is known as soft covering approximation space (SCAS).

*Definition 4* (Zhan and Wang, 2019). Let $S = (\Omega, \tilde{C}_K)$ be a SCAS. For each $v_i \in \Omega$, the soft adhesion of $v_i$ are defined as $SA(v_i) = \{v_k \in \Omega: \forall\ b_j \in B(v_i \in \widetilde{N}(b_j) \leftrightarrow v_k \in \widetilde{N}(b_j))\}$, where $v_i, v_k \in \Omega$.

*Definition 5* (Zhan and Wang, 2019). Let $S = (\Omega, \tilde{C}_K)$ be a SCAS. For each $M \subseteq \Omega$, the soft covering lower approximation (SCLA) and soft covering upper approximation (SCUA) are respectively defined as $\underline{SC}(M) = \{v_i \in \Omega: SA(v_i) \subseteq M\}$ and $\overline{SC}(M) = \{v_i \in \Omega: SA(v_i) \cap M \neq \emptyset\}$. If $\underline{SC}(M) \neq \overline{SC}(M)$, then $M$ is known as Z-soft covering based rough set. Moreover, the sets

$Pos(M) = \underline{SC}(M)$, $Neg(M) = \Omega - \overline{SC}(M)$, and $Bnd(M) = \overline{SC}(M) - \underline{SC}(M)$ are known as the Z-soft positive region, Z-soft negative region and Z- soft boundary region of $M$ respectively.

*Definition 6* (Zadeh, 1965). Let $F(\Omega)$ indicates the family of all fuzzy subsets of $\Omega$. Let $N$ is a mapping defined by $N: B \to F(\Omega)$, then the ordered pair $(N, B)$ is known as fuzzy soft set (FSS) over $\Omega$.

*Definition 7* (Cagman and Enginoglu, 2012). Let $(n_B, E)$ be a FSS over $\Omega$. Then, a subset of $\Omega \times E$ is uniquely defined by $R_B = \{(v_i, b_j): b_j \in B, v_i \in n_B(b)\}$ which is known as the relation form of $(n_B, E)$. The indicator function of $R_B$ is given by $\chi_{R_B}: \Omega \times E \to [0,1]$, $\chi_{R_B}(v_i, b_j) = \mu(v_i, b_j)$, where $\mu(v_i, b_j)$ is the membership value of

$v_i$ with respect to $b_j$. If $\Omega = \{v_1, v_2, \ldots, v_p\}$ and $B = \{b_1, b_2, \ldots, b_q\}$, then $R_B$ can be written in the table form as below,

| $R_B$ | $b_1$ | $b_2$ | $\cdots$ | $b_q$ |
|---|---|---|---|---|
| $v_1$ | $\chi_{R_B}(v_1, b_1)$ | $\chi_{R_B}(v_1, b_2)$ | $\cdots$ | $\chi_{R_B}(v_1, b_q)$ |
| $v_2$ | $\chi_{R_B}(v_2, b_1)$ | $\chi_{R_B}(v_2, b_2)$ | $\cdots$ | $\chi_{R_B}(v_2, b_q)$ |
| $\vdots$ | $\cdots$ | $\cdots$ | $\ddots$ | $\cdots$ |
| $v_p$ | $\chi_{R_B}(v_p, b_1)$ | $\chi_{R_B}(v_p, b_2)$ | $\cdots$ | $\chi_{R_B}(v_p, b_q)$ |

If $m_{ij} = \chi_{R_B}(v_i, b_j)$, we define a matrix $[m_{ij}]_{p\times q} = \begin{pmatrix} m_{11} & \ldots & m_{1q} \\ \vdots & \ddots & \vdots \\ m_{p1} & \ldots & m_{pq} \end{pmatrix}$, which is known as fuzzy soft matrix of $(n_B, E)$ of order $p \times q$ over Ω.

*Definition 8* (Zhang et al., 2004). If $(\bigcup_{b_j \in B} N(b_j))(v_i) = 1$, for all $v_i \in \Omega$, then $(N, B)$ is known as fuzzy soft covering over $\Omega$. Then $(\Omega, C_K)$ is known as fuzzy soft covering approximation space (FSCAS).

*Definition 9* (Zhang and Zhan, 2019). Let $(\Omega, C_K)$ be a FSCAS. If $(\bigcup_{b_j \in B} N(b_j))(v_i) \geq \beta$, for all $v_i \in \Omega$, then $\mathfrak{S} = (N, B)$ is known as fuzzy soft $\beta$-covering set (FS$\beta$CS) over $\Omega$. Then $(\Omega, C_K)$ is known as fuzzy soft $\beta$-covering approximation space (FS$\beta$CAS).

## 3. Generalized Z-fuzzy soft $\beta$-covering based rough matrices

In this section, we propose a Z-fuzzy soft $\beta$-covering based rough set with respect to $\beta$-level soft set and a new type of matrix is introduced by using Z-fuzzy soft $\beta$-covering based rough set namely, Z-fuzzy soft $\beta$-covering based rough (Z-FS$\beta$CR) matrix. As a generalization of this concept, generalized Z-fuzzy soft $\beta$-covering based rough (Z-GFS$\beta$CR) matrix is proposed.

*Definition 10.* Let $\mathfrak{S} = (N, B)$ be a FS$\beta$CS over $\Omega$. Let $\beta \in (0,1]$. The $\beta$-level soft set of $(N, B)$ is a crisp soft set defined by $L((N, B), \beta) =$

$$N_\beta(b_j)(v_i) = \begin{cases} 1, & \text{if } N(b_j)(v_i) \ \geq \ \beta \,; \\ 0, & \text{if } N(b_j)(v_i) \ < \ \beta \,. \end{cases}$$

, where $i = 1, \ldots, p$ and $j = 1, \ldots, q$.

*Definition 11.* Let $(\Omega, C_K)$ be a FS$\beta$CAS. For each $v_i \in \Omega$, the fuzzy soft $\beta$-adhesion of $v_i$ are defined as $SA_\beta(v_i) = \{v_k \in \Omega \colon \forall\ b_j \in B(v_i \in N_\beta(b_j) \leftrightarrow v_k \in N_\beta(b_j))\}$.

*Definition 12.* Let $\mathfrak{S} = (N, B)$ be a FS$\beta$CS over $\Omega$ and $L$ be a $\beta$-level soft set of $\mathfrak{S}$. Then $\mathcal{J} = (\Omega, L)$ is known as fuzzy soft $\beta$-covering approximation space (FS$\beta$CAS) with respect to $\beta$-level soft set of $\mathfrak{S}$. For all $M \subseteq \Omega$, the fuzzy soft $\beta$-covering lower approximation (FS$\beta$CLA) and fuzzy soft $\beta$-covering upper approximation (FS$\beta$CUA) are defined as, $\underline{FS}(M) = \{v_i \in \Omega \colon SA_\beta(v_i) \subseteq M\}$ and $\overline{FS}(M) = \{v_i \in \Omega \colon SA_\beta(v_i) \cap M \neq \emptyset\}$ respectively . If $\underline{FS}(M) \neq \overline{FS}(M)$, then $M$ is called Z-fuzzy soft $\beta$-covering based rough set (Z-FS$\beta$CRS). Moreover, the sets $Pos(M) = \underline{FS}(M)$, $Neg(M) = \Omega - \overline{FS}(M)$

and $Bnd(M) = \overline{FS}(M) - \underline{FS}(M)$ are known as the Z-fuzzy soft $\beta$-positive region, Z-fuzzy soft $\beta$-negative region and Z-fuzzy soft $\beta$-boundary region of $M$ respectively.

*Lemma 1.* If $M = \emptyset$, then $\underline{FS}(M) = \emptyset$, $\overline{FS}(M) = \emptyset$, $Pos(M) = \emptyset$, $Neg(M) = \Omega$ and $Bnd(M) = \emptyset$. If $M = \Omega$, then $\underline{FS}(M) = \Omega$, $\overline{FS}(M) = \Omega$, $Pos(M) = \Omega$, $Neg(M) = \emptyset$ and $Bnd(M) = \emptyset$.

The following example proves that if $M \subseteq O$, then Pos(M) ⊂ Pos(O), Neg(M) ⊃ Neg(O) but Bnd(M) need not to be a subset of Bnd(O).

*Example 1.* A university is conducting an interview for an assistant professor job. The candidates who have applied for the interview form a set $\Omega = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and the attribute set includes their important features such as communication skills $(b_1)$, teaching skills $(b_2)$, academic records $(b_3)$ and experience $(b_4)$. Let $(N, B)$ be the FS$\beta$CS over $\Omega$ in Table 1. Let $N(b_1) = \{v_1, v_2, v_3, v_4, v_6\}$, $N(b_2) = \{v_2, v_6\}$, $N(b_3) = \{v_1, v_3, v_4, v_5\}$, $N(b_4) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ *and* $N(b_5) = \{v_5\}$. *Then the fuzzy soft $\beta$-adhesion are* $SA_\beta(v_1) = \{v_1, v_3, v_4\}$, $SA_\beta(v_2) = \{v_2, v_6\}$, $SA_\beta(v_3) = \{v_1, v_3, v_4\}$, $SA_\beta(v_4) = \{v_1, v_3, v_4\}$, $SA_\beta(v_5) = \{v_5\}$ *and* $SA_\beta(v_6) = \{v_2, v_6\}$. *Let* $M = \{v_2, v_5\}$, $O = \{v_2, v_3, v_4, v_5, v_6\}$ *and* $P = \{v_1, v_2, v_4, v_6\}$ *are the subsets of* $\Omega$.

For $M$, $\underline{FS}(M) = \{v_5\}$, $\overline{FS}(M) = \{v_2, v_5, v_6\}$, $Pos(M) = \{v_5\}$, $Neg(M) = \{v_1, v_3, v_4\}$ and $Bnd(M) = \{v_2, v_6\}$.

For $O$, $\underline{FS}(O) = \{v_2, v_5, v_6\}$, $\overline{FS}(O) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $Pos(O) = \{v_2, v_5, v_6\}$, $Neg(O) = \emptyset$ and $Bnd(O) = \{v_1, v_3, v_4\}$.

For $P$, $\underline{FS}(P) = \{v_2, v_6\}$, $\overline{FS}(P) = \{v_1, v_2, v_3, v_4, v_6\}$, $Pos(P) = \{v_2, v_6\}$, $Neg(P) = \{v_5\}$ and $Bnd(P) = \{v_1, v_3, v_4\}$.

Here, $M \subseteq O$ but $Bnd(M) \nsubseteq Bnd(O)$.

*Theorem 1.* Let $(\Omega, L)$ be a fuzzy soft $\beta$-covering approximation space and $M, O \subseteq \Omega$. Then the fuzzy soft $\beta$-covering lower approximation and fuzzy soft $\beta$-covering upper approximations satisfies the following properties:

1) $\underline{FS}(M) \subseteq M \subseteq \overline{FS}(M)$.
2) $\underline{FS}(\emptyset) = \emptyset = \overline{FS}(\emptyset)$.
3) $\underline{FS}(\Omega) = \Omega = \overline{FS}(\Omega)$.
4) $\overline{FS}(M \cup O) = \overline{FS}(M) \cup \overline{FS}(O)$.
5) $\underline{FS}(M \cap O) = \underline{FS}(M) \cap \underline{FS}(O)$.
6) If $M \subseteq O$, then $\underline{FS}(M) \subseteq \underline{FS}(O)$.
7) If $M \subseteq O$, then $\overline{FS}(M) \subseteq \overline{FS}(O)$.
8) $\underline{FS}(-M) = -\overline{FS}(M)$.
9) $\overline{FS}(-M) = -\underline{FS}(M)$.
10) $\underline{FS}(\underline{FS}(M)) = \underline{FS}(M)$.
11) $\overline{FS}(\overline{FS}(M)) = \overline{FS}(M)$.

Proof.

1) If $v_i \in \underline{FS}(M)$ then $SA(v_i) \subseteq M$ since $v_i \in SA(v_i)$ hence $v_i \in M$ and $\underline{FS}(M) \subseteq M$. If $v_i \in M$ then $SA(v_i) \cap M \neq \emptyset$, since $v_i \in SA(v_i) \cap M$. Hence $v_i \in \overline{FS}(M)$ and $M \subseteq \overline{FS}(M)$.

2) From 1), $\underline{FS}(\emptyset) \subseteq \emptyset$ and $\emptyset \subseteq \underline{FS}(\emptyset)$, thus $\underline{FS}(\emptyset) = \emptyset$. Consider $\overline{FS}(\emptyset) \neq \emptyset$. Then there exist $v_i$ such that $v_i \in \overline{FS}(\emptyset)$. Hence $SA(v_i) \cap \emptyset \neq \emptyset$, but $SA(v_i) \cap \emptyset = \emptyset$. This contradicts our assumption. Thus, $\overline{FS}(\emptyset) = \emptyset$.

3) From 1), $\underline{FS}(\Omega) \subseteq \Omega$. Now we have to prove that $\Omega \subseteq \underline{FS}(\Omega)$. If $v_i \in \Omega$ then $SA(v_i) \subseteq \Omega$. Hence $v_i \in \underline{FS}(\Omega)$, thus $\underline{FS}(\Omega) = \Omega$. From 1), $\Omega \subseteq \overline{FS}(\Omega)$ and $\overline{FS}(\Omega) \subseteq \Omega$, thus $\overline{FS}(\Omega) = \Omega$.

4) $v_i \in \overline{FS}(M \cup O)$ iff $SA(v_i) \cap (M \cup O) \neq \emptyset$ iff $SA(v_i) \cap M \cup SA(v_i) \cap O \neq \emptyset$ iff $SA(v_i) \cap M \neq \emptyset$ and $SA(v_i) \cap M \neq \emptyset$ iff $v_i \in \overline{FS}(M)$ and $v_i \in \overline{FS}(O)$ iff $v_i \in \overline{FS}(M) \cup \overline{FS}(O)$. Thus, $\overline{FS}(M \cup O) = \overline{FS}(M) \cup \overline{FS}(O)$.

5) $v_i \in \underline{FS}(M \cap O)$ iff $SA(v_i) \subseteq M \cap O$ iff $SA(v_i) \subseteq M$ or $SA(v_i) \subseteq O$ iff $v_i \in \underline{FS}(M) \cap \underline{FS}(O)$.

6) Let $v_i \in \underline{FS}(M)$, by Definition 12, $SA(v_i) \subseteq M$ this implies $SA(v_i) \subseteq M \subset O$. Hence $v_i \in SA(v_i) \subseteq O$ implies $v_i \in \underline{FS}(O)$. Thus, $\underline{FS}(M) \subseteq \underline{FS}(O)$.

7) Since $M \subseteq O$ iff $M \cup O$, hence $\overline{FS}(M \cup O) = \overline{FS}(O)$ and by using 4) we get $\overline{FS}(M) \cup \overline{FS}(O) = \overline{FS}(O)$. Hence $\overline{FS}(M) \subseteq \overline{FS}(O)$.

8) If $v_i \in \underline{FS}(M)$ then $SA(v_i) \subseteq M$ iff $SA(v_i) \cap -M = \emptyset$ iff $SA(v_i) \notin \overline{FS}(-M)$ iff $v_i \in -\overline{FS}(-M)$. Hence $\underline{FS}(M) = -\overline{FS}(-M)$.

9) By replacing $M$ by $-M$ in the proof of 10), we get $\overline{FS}(-M) = -\underline{FS}(M)$.

10) From 1), $\underline{FS}(\underline{FS}(M)) \subseteq \underline{FS}(M)$. Now we have to prove that $\underline{FS}(M) \subseteq \underline{FS}(\underline{FS}(M))$. If $v_i \in \underline{FS}(M)$ then $SA(v_i) \subseteq M$, hence $\underline{FS}(SA(v_i)) \subseteq \underline{FS}(M)$ but $\underline{FS}(SA(v_i)) = SA(v_i)$, thus $SA(v_i) \subseteq \underline{FS}(M)$ and $v_i \in \underline{FS}(\underline{FS}(M))$. Hence $\underline{FS}(M) \subseteq \underline{FS}(\underline{FS}(M))$.

11) From 1), $\overline{FS}(M) \subseteq \overline{FS}(\overline{FS}(M))$. Now we have to prove $\overline{FS}(\overline{FS}(M)) \subseteq \overline{FS}(M)$. If $v_i \in \overline{FS}(\overline{FS}(M))$, then $SA(v_i) \cap \overline{FS}(M) \neq \emptyset$, but $SA(v_i) = SA(v_k)$, thus $SA(v_i) \cap M \neq \emptyset$.

The above theorem shows that the fuzzy soft $\beta$-covering lower and upper approximations satisfy the Pawlak's rough lower and upper approximations properties. Hence, the Z-fuzzy soft $\beta$-covering based rough set is significant.

*Definition 13.* Let $M \subseteq \Omega$ be a Z-FS$\beta$CRS with Z-fuzzy soft $\beta$-positive region, Z-fuzzy soft $\beta$-negative region and Z-fuzzy soft $\beta$-boundary region. Define a function $I_{FS\beta CR}: \Omega \rightarrow \{0, 0.5, 1\}$ such that

$$I_{FS\beta CR}(v_i) = \begin{cases} 1, & \text{if } v_i \in Pos(M)\,; \\ 0, & \text{if } v_i \in Neg(M)\,; \\ 0.5, & \text{if } v_i \in Bnd(M)\,. \end{cases}$$

The matrix formed by $I_{FS\beta CR}$ is defined as $[m_{ij}]_{p\times q} = \begin{pmatrix} m_{11} & \cdots & m_{1q} \\ \vdots & \ddots & \vdots \\ m_{p1} & \cdots & m_{pq} \end{pmatrix}$, where $m_{ij} \in \{0,0.5,1\}$ and it is known as Z-fuzzy soft $\beta$-covering based rough (Z-FS$\beta$CR) matrix over $\Omega$ of order $p \times q$. The family of all Z-FS$\beta$CR matrices over $\Omega$ of order $p \times q$ are denoted by Z-FS$\beta$CR $_{p\times q}$.

*Definition 14*. Let $M \subseteq \Omega$ be a Z-FS$\beta$CRS with the Z-fuzzy soft $\beta$-positive, Z-fuzzy soft $\beta$-negative and Z fuzzy soft $\beta$-boundary regions. Define a function $I_{GFS\beta CR}: \Omega \to [0,1]$ such that

$$I_{GFS\beta CR}(v_i) = \begin{cases} 1, & \text{if } v_i \in Pos(M)\,; \\ 0, & \text{if } v_i \in Neg(M)\,; \\ a, & \text{if } v_i \in Bnd(M), \quad a \in (0,1)\,. \end{cases}$$

The matrix formed from $I_{GFS\beta CR}$ is defined as $[m_{ij}]_{p\times q} = \begin{pmatrix} m_{11} & \cdots & m_{1q} \\ \vdots & \ddots & \vdots \\ m_{p1} & \cdots & m_{pq} \end{pmatrix}$, where $m_{ij} \in [0,1]$, and it is known as generalized Z-fuzzy soft $\beta$-covering based rough (Z-GFS$\beta$CR) matrix over $\Omega$ of order $p \times q$. The family of all Z-GFS$\beta$CR matrices over $\Omega$ of order $p \times q$ are denoted by Z-GFS$\beta$CR $_{p\times q}$.

*Example 2*. (Continued from *Example 1*) Let $(N, B)$ be the FS$\beta$CS over $\Omega$ in Table 1. Let $\beta$ = 0.5. Thus, the $\beta$-level soft set of $(N, B)$ is shown in Table 2. The ordered pair $\mathcal{J}$ = $(\Omega, L)$ be the FS$\beta$CAS. Then the fuzzy soft $\beta$-adhesion with respect to $\beta$-level soft set are $SA_\beta(v_1) = \{v_1, v_4\}$, $SA_\beta(v_2) = \{v_2, v_5\}$, $SA_\beta(v_3) = \{v_3\}$, $SA_\beta(v_4) = \{v_1, v_4\}$, $SA_\beta(v_5) = \{v_2, v_5\}$ and $SA_\beta(v_6) = \{v_6\}$. For $M = \{v_1, v_3, v_5\} \subseteq \Omega$, then we get $\underline{FS}(M) = \{v_3\}$, $\overline{FS}(M) = \{v_1, v_2, v_3, v_4, v_5\}$, $Pos(M) = \{v_3\}$, $Neg(M) = \{v_6\}$ and $Bnd(M) = \{v_1, v_2, v_4, v_5\}$. Here $\underline{FS}(M) \neq \overline{FS}(M)$, thus $M$ is known as Z-fuzzy soft $\beta$-covering based rough set. The Z-GFS$\beta$CR matrix of order $6 \times 4$ is given by

$$[m_{ij}]_{6\times 4} = \begin{pmatrix} a & a & a & a \\ a & a & a & a \\ 1 & 1 & 1 & 1 \\ a & a & a & a \\ a & a & a & a \\ 0 & 0 & 0 & 0 \end{pmatrix}, \text{where } a \in (0,1).$$

**Table 1.** Tabular representation for (*N, B*)

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|
| $v_1$ | 0.4 | 0.5 | 0.6 | 0.7 |
| $v_2$ | 0.3 | 0.2 | 0.8 | 0.7 |
| $v_3$ | 0.5 | 0.1 | 0.6 | 0.4 |
| $v_4$ | 0.4 | 0.5 | 0.7 | 0.6 |
| $v_5$ | 0.4 | 0.3 | 0.7 | 0.6 |
| $v_6$ | 0.7 | 0.8 | 0.3 | 0.2 |

**Table 2.** Tabular representation of $\beta$-level soft set $L((N, B), 0.5)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|
| $v_1$ | 0 | 1 | 1 | 1 |
| $v_2$ | 0 | 0 | 1 | 1 |
| $v_3$ | 1 | 0 | 1 | 0 |
| $v_4$ | 0 | 1 | 1 | 1 |
| $v_5$ | 0 | 0 | 1 | 1 |
| $v_6$ | 1 | 1 | 0 | 0 |

*Definition 15.* Let $[m_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. If there is only one element in the attribute set, then $[m_{ij}]$ is known as Z-fuzzy soft $\beta$-covering based row matrix and denoted by $[m_{i1}]$.

*Definition 16.* Let $[m_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. If there is only one element in the universal set, then $[m_{ij}]$ is known as Z-fuzzy soft $\beta$-covering based column matrix and denoted by $[m_{1j}]$.

*Definition 17.* Let $[m_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. For each $i$ and $j$, if $m_{ij} = 0$, for all $v_i \in Pos(M)$ then $[m_{ij}]$ is known as Zero Z-fuzzy soft $\beta$-covering based rough matrix and denoted by [0].

*Definition 18.* Let $[m_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. For each $i$ and $j$, if $m_{ij} = 1$, for all $v_i \in Neg(M)$ then $[m_{ij}]$ is known as Universal Z-fuzzy soft $\beta$-covering based rough matrix and denoted by [1].

*Definition 19.* Let $[m_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. For each $i$ and $j$, if $m_{ij} = a$, for all $v_i \in Bnd(M)$, $a \in (0,1)$ then $[m_{ij}]$ is known as Universal generalized Z-fuzzy soft $\beta$-covering based rough matrix and denoted by $[a]$.

*Definition 20.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. If $m_{ij} \leq n_{ij}$ for each $i$ and $j$ then $[m_{ij}]$ is a Z-fuzzy soft $\beta$-covering based rough sub matrix of $[n_{ij}]$ and denoted by $[m_{ij}] \tilde{\subseteq} [n_{ij}]$.

*Definition 21.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. If $m_{ij} \leq n_{ij}$ for at least one element $m_{ij} < n_{ij}$ for each $i$ and $j$ then $[m_{ij}]$ is a proper Z-fuzzy soft $\beta$-covering based rough sub matrix of $[n_{ij}]$ and denoted by $[m_{ij}] \tilde{\subset} [n_{ij}]$.

*Definition 22.* If $m_{ij} = n_{ij}$ for each $i$ and $j$ then $[m_{ij}]$ and $[n_{ij}]$ are called Z-fuzzy soft $\beta$-covering based rough equal matrices and denoted by $[m_{ij}] = [n_{ij}]$.

*Definition 23.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. The union of $[m_{ij}]$ and $[n_{ij}]$ is denoted by $[m_{ij}] \tilde{\cup} [n_{ij}]$, defined as $[k_{ij}]$ where $k_{ij} = max\{m_{ij}, n_{ij}\}$, for each $i$ and $j$.

*Definition 24.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. The intersection of $[m_{ij}]$ and $[n_{ij}]$ is denoted by $[m_{ij}] \tilde{\cap} [n_{ij}]$, defined as $[k_{ij}]$ where $k_{ij} = min\{m_{ij}, n_{ij}\}$, for each $i$ and $j$.

*Definition 25.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. The complement of $[m_{ij}]$ is denoted by $[m_{ij}]^c$ defined as $[k_{ij}]$ where $k_{ij} = 1 - m_{ij}$ for each $i$ and $j$.

*Definition 26.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. The matrices $[m_{ij}]$ and $[n_{ij}]$ are disjoint if $[m_{ij}] \,\tilde{\cap}\, [n_{ij}]$ = [0] for each $i$ and $j$.

*Proposition 1.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then,

1) $[[m_{ij}]^c]^c = [m_{ij}]$.
2) $[0]^c = [1]$.

*Proof.*
The proof is obvious from Definition 25.

*Proposition 2.* Let $[m_{ij}], [n_{ij}], [k_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then,

1) $[m_{ij}] \,\tilde{\subseteq}\, [1]$.
2) $[0] \,\tilde{\subseteq}\, [m_{ij}]$.
3) $[m_{ij}] \,\tilde{\subseteq}\, [m_{ij}]$.
4) $[m_{ij}] \,\tilde{\subseteq}\, [n_{ij}]$ and $[n_{ij}] \,\tilde{\subseteq}\, [k_{ij}]$ implies $[m_{ij}] \,\tilde{\subseteq}\, [k_{ij}]$.
5) $[m_{ij}] \,\tilde{\subseteq}\, [n_{ij}]$ and $[n_{ij}] \,\tilde{\subseteq}\, [m_{ij}]$ if and only if $[m_{ij}] = [n_{ij}]$.
6) $[m_{ij}] = [n_{ij}]$ and $[n_{ij}] = [k_{ij}]$ if and only if $[m_{ij}] = [k_{ij}]$.

*Proof.*
The proof is obvious from Definitions 20 and 22.

*Proposition 3.* Let $[m_{ij}], [n_{ij}], [k_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then,

1) $[m_{ij}] \,\tilde{\cup}\, [m_{ij}] = [m_{ij}]$.
2) $[m_{ij}] \,\tilde{\cup}\, [0] = [m_{ij}]$.
3) $[m_{ij}] \,\tilde{\cup}\, [1] = [1]$.
4) $[m_{ij}] \,\tilde{\cup}\, [n_{ij}] = [n_{ij}] \,\tilde{\cup}\, [m_{ij}]$.
5) $[m_{ij}] \,\tilde{\cup}\, ([n_{ij}] \,\tilde{\cup}\, [k_{ij}]) = ([m_{ij}] \,\tilde{\cup}\, [n_{ij}]) \,\tilde{\cup}\, [k_{ij}]$.

*Proof.*
The proof is obvious from Definition 23.

*Proposition 4.* Let $[m_{ij}], [n_{ij}], [k_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then,

1) $[m_{ij}] \,\tilde{\cap}\, [m_{ij}] = [m_{ij}]$.
2) $[m_{ij}] \,\tilde{\cap}\, [0] = [0]$.
3) $[m_{ij}] \,\tilde{\cap}\, [1] = [m_{ij}]$.
4) $[m_{ij}] \,\tilde{\cap}\, [n_{ij}] = [n_{ij}] \,\tilde{\cap}\, [m_{ij}]$.
5) $[m_{ij}] \,\tilde{\cap}\, ([n_{ij}] \,\tilde{\cap}\, [k_{ij}]) = ([m_{ij}] \,\tilde{\cap}\, [n_{ij}]) \,\tilde{\cap}\, [k_{ij}]$.

*Proof.*
The proof is obvious from Definition 24.

*Proposition 5.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then, De Morgan's inclusions are true.

1) $([m_{ij}] \,\tilde{\cup}\, [n_{ij}])^c \,\tilde{\subseteq}\, [m_{ij}]^c \,\tilde{\cap}\, [n_{ij}]^c$.
2) $([m_{ij}] \,\tilde{\cap}\, [n_{ij}])^c \,\tilde{\subseteq}\, [m_{ij}]^c \,\tilde{\cup}\, [n_{ij}]^c$.

*Proof.*
The proof is obvious from Definitions 20, 23 and 24.

*Proposition 6.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then, De Morgan's laws are true.

1) $\left([m_{ij}] \tilde{\cup} [n_{ij}]\right)^c = [m_{ij}]^c \tilde{\cap} [n_{ij}]^c$.
2) $\left([m_{ij}] \tilde{\cap} [n_{ij}]\right)^c = [m_{ij}]^c \tilde{\cup} [n_{ij}]^c$.

*Proof.*

1) For all $i$ and $j$,

$$\begin{aligned}\left([m_{ij}] \tilde{\cup} [n_{ij}]\right)^c &= \left[max\{m_{ij}, n_{ij}\}\right]^c\\ &= 1 - max\{m_{ij}, n_{ij}\}\\ &= min\{1 - m_{ij}, 1 - n_{ij}\}\\ &= [m_{ij}]^c \tilde{\cap} [n_{ij}]^c.\end{aligned}$$

2) It is similar to the proof 1).

*Proposition 7.* Let $[m_{ij}], [n_{ij}], [k_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. Then,

1) $[m_{ij}] \tilde{\cup} \left([n_{ij}] \tilde{\cap} [k_{ij}]\right) = \left([m_{ij}] \tilde{\cup} [n_{ij}]\right) \tilde{\cap} \left([m_{ij}] \tilde{\cup} [k_{ij}]\right)$.
2) $[m_{ij}] \tilde{\cap} \left([n_{ij}] \tilde{\cup} [k_{ij}]\right) = \left([m_{ij}] \tilde{\cap} [n_{ij}]\right) \tilde{\cup} \left([m_{ij}] \tilde{\cap} [k_{ij}]\right)$.

*Proof.*

1) For all $i$ and $j$,

$$\begin{aligned}[m_{ij}] \tilde{\cup} \left([n_{ij}] \tilde{\cap} [k_{ij}]\right) &= max\{m_{ij}, \left([n_{ij}] \tilde{\cap} [k_{ij}]\right)\}\\ &= max\{m_{ij}, \left(min\{n_{ij}, k_{ij}\}\right)\}\\ &= min\left\{max\{m_{ij}, n_{ij}\}, max\{m_{ij}, k_{ij}\}\right\}\\ &= min\{\left([m_{ij}] \tilde{\cup} [n_{ij}]\right), \left([m_{ij}] \tilde{\cup} [k_{ij}]\right)\}\\ &= \left([m_{ij}] \tilde{\cup} [n_{ij}]\right) \tilde{\cap} \left([m_{ij}] \tilde{\cup} [k_{ij}]\right).\end{aligned}$$

2) It is similar to the proof 1).

*Definition 27.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. The not union of $[m_{ij}]$ and $[n_{ij}]$ is defined as $[k_{ij}]$, where $k_{ij} = max\{1 - m_{ij}, 1 - n_{ij}\}$ for each $i$ and $j$, and it is denoted by $[m_{ij}] \overline{\tilde{\cup}} [n_{ij}]$.

*Definition 28.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. The not intersection of $[m_{ij}]$ and $[n_{ij}]$ is defined as $[k_{ij}]$, where $k_{ij} = min\{1 - m_{ij}, 1 - n_{ij}\}$, for each $i$ and $j$, and it is denoted by $[m_{ij}] \overline{\tilde{\cap}} [n_{ij}]$.

*Proposition 8.* Let $[m_{ij}], [n_{ij}], [k_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. Then,

1) $[m_{ij}] \overline{\tilde{\cup}} [m_{ij}] = [m_{ij}]^c$.
2) $[m_{ij}] \overline{\tilde{\cup}} [0] = [1]$.
3) $[m_{ij}] \overline{\tilde{\cup}} [1] = [m_{ij}]^c$.
4) $[m_{ij}] \overline{\tilde{\cup}} [n_{ij}] = [n_{ij}] \overline{\tilde{\cup}} [m_{ij}]$.
5) $[m_{ij}] \overline{\tilde{\cup}} \left([n_{ij}] \overline{\tilde{\cup}} [k_{ij}]\right) = \left([m_{ij}] \overline{\tilde{\cup}} [n_{ij}]\right) \overline{\tilde{\cup}} [k_{ij}]$.

*Proof.*

The proof is obvious from Definitions 22, 25 and 27.

*Proposition 9.* Let $[m_{ij}], [n_{ij}], [k_{ij}] \in$ Z-GFS$\beta$CR$_{p\times q}$. Then,

1) $[m_{ij}] \overline{\tilde{\cap}} [m_{ij}] = [m_{ij}]^c$.
2) $[m_{ij}] \overline{\tilde{\cap}} [0] = [m_{ij}]^c$.

3) $[m_{ij}] \,\overline{\widetilde{\cap}}\, [1] = [0]$.

4) $[m_{ij}] \,\overline{\widetilde{\cap}}\, [n_{ij}] = [n_{ij}] \,\overline{\widetilde{\cap}}\, [m_{ij}]$.

5) $[m_{ij}] \,\overline{\widetilde{\cap}}\, \left([n_{ij}] \,\overline{\widetilde{\cap}}\, [k_{ij}]\right) = \left([m_{ij}] \,\overline{\widetilde{\cap}}\, [n_{ij}]\right) \overline{\widetilde{\cap}}\, [k_{ij}]$.

*Proof.*

The proof is obvious from Definition 22, 25 and 28.

*Proposition 10.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then, De Morgan's inclusions are true.

1) $\left([m_{ij}] \,\overline{\widetilde{\cup}}\, [n_{ij}]\right)^c \,\widetilde{\subseteq}\, [m_{ij}]^c \,\overline{\widetilde{\cap}}\, [n_{ij}]^c$.

2) $\left([m_{ij}] \,\overline{\widetilde{\cap}}\, [n_{ij}]\right)^c \,\widetilde{\subseteq}\, [m_{ij}]^c \,\overline{\widetilde{\cup}}\, [n_{ij}]^c$.

*Proof.*

The proof is obvious from Definitions 20, 27 and 28.

*Proposition 11.* Let $[m_{ij}], [n_{ij}] \in$ Z-GFS$\beta$CR $_{p\times q}$. Then, De Morgan's laws are true.

1) $\left([m_{ij}] \,\overline{\widetilde{\cup}}\, [n_{ij}]\right)^c = [m_{ij}]^c \,\overline{\widetilde{\cap}}\, [n_{ij}]^c$.

2) $\left([m_{ij}] \,\overline{\widetilde{\cap}}\, [n_{ij}]\right)^c = [m_{ij}]^c \,\overline{\widetilde{\cup}}\, [n_{ij}]^c$.

*Proof.*

1) For each $i$ and $j$,

$$\begin{aligned}\left([m_{ij}] \,\overline{\widetilde{\cup}}\, [n_{ij}]\right)^c &= \left[max\{1-m_{ij}, 1-n_{ij}\}\right]^c \\ &= 1 - max\{1-m_{ij}, 1-n_{ij}\} \\ &= min\{1-(1-m_{ij}), 1-(1-n_{ij})\} \\ &= min\{1-m_{ij}^c, 1-n_{ij}^c\} \\ &= [m_{ij}]^c \,\widetilde{\cap}\, [n_{ij}]^c.\end{aligned}$$

2) It is similar to the proof 1).

## 4. A novel approach to MAGDM using Z-GFS$\beta$CR matrices

In this section, a decision-making algorithm is created to select the best object from a list of possible objects $\Omega$ based on a decision maker's chosen parameters.

### 4.1. Description and Process

Let $\Omega = \{v_1, v_2, \ldots, v_p\}$ be $p$ alternatives and $E = \{b_1, b_2, \ldots, b_q\}$ be the set of all attributes.

*Step 1:* Choose the appropriate subsets of the attribute set $E$ and construct the fuzzy soft $\beta$-covering set $(N_l, B_l)$ over $\Omega$, where $l = 1,2,\ldots,k$, for each attribute sets. Define a matrix $[m_{ij}]_{p\times q} = \begin{pmatrix} m_{11} & \cdots & m_{1q} \\ \vdots & \ddots & \vdots \\ m_{p1} & \cdots & m_{pq} \end{pmatrix}$, which is known as the Z-fuzzy soft $\beta$-covering rough matrix of $(N_l, B_l)$ of order $p \times q$ over $\Omega$.

*Step 2:* Let $(N_l, B_l)$ be a FS$\beta$CS over $\Omega$. Let $\beta$ = 0.5. Compute the $\beta$-level soft set of $(N_l, B_l)$ by the formula,

$$N_\beta(b_j)(v_i) = \begin{cases} 1, & \text{if } N(b_j)(v_i) \ \geq \ \beta\,; \\ 0, & \text{if } N(b_j)(v_i) \ < \ \beta\,. \end{cases}$$

Then the ordered pair $\mathcal{J} = (\Omega, L_l)$ is the FS$\beta$CAS with respect to $\beta$-level soft set of each $(N_l, B_l)$ are computed.

*Step 3:* Using $\beta$-level soft set, calculate the fuzzy soft $\beta$-adhesion of each $v_i \in \Omega$. Now compute the FS$\beta$CLA and FS$\beta$CUA by using the formula, $\underline{FS}(M) = \{v_i \in \Omega: SA_\beta(v_i) \subseteq M\}$ and $\overline{FS}(M) = \{v_i \in \Omega: SA_\beta(v_i) \cap M \neq \emptyset\}$. By means of FS$\beta$CLA and FS$\beta$CUA , calculate the sets, $Pos(M) = \underline{FS}(M)$, $Neg(M) = \Omega - \overline{FS}(M)$, and $Bnd(M) = \overline{FS}(M) - \underline{FS}(M)$.

*Step 4:* By using Definition 14, construct the Z-GFS$\beta$CR matrices of order $p \times q$ over $\Omega$.

*Step 5:* By using Saaty's (2008) nine-point scale, construct the pairwise comparison matrices for each criteria according to the three experts. Calculate the weight for each criteria using AHP method.

*Step 6:* The calculated weight $[W_{B_l}]$ are multiplied with its corresponding Z-GFS$\beta$CR matrix and it is denoted by $[D_{B_l}]$, where $[m_{ij}] \times [W_{B_l}] = [D_{B_l}]$.

*Step 7:* Select the value for $\alpha$ and by means of max-min technique, determine the best alternative from the list of $\Omega$.

### 4.2. Algorithm

- Construct the fuzzy soft $\beta$-covering set $(N_l, B_l)$ based on the important parameters.
- Compute the $\beta$-level soft set.
- Calculate $\underline{FS}(M)$ and $\overline{FS}(M)$ by using fuzzy soft $\beta$-adhesion and find Pos(M), Neg(M) and Bnd(M).
- Create the Z-GFS$\beta$CR matrix.
- Compute the weight for each criteria by means of AHP method with the help of experts (interviewers).
- Multiply each $[W_{B_l}]$ with the corresponding Z-GFS$\beta$CR matrices.
- By means of max-min technique, determine the best alternative from $\Omega$.

### 4.3. Illustrative Example

The steps mentioned in the algorithm are demonstrated in the following numerical example.

*Example 3.* A university is conducting an interview for an assistant professor job. The candidates who have applied for the interview form a set $\Omega = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and the attribute set includes their important features such as teaching skill $(b_1)$, experience $(b_2)$, presentation skill $(b_3)$, communication skill $(b_4)$, academic records $(b_5)$, time management Skills $(b_6)$, and patience $(b_7)$.

*Step 1:* Let the choice parameters of the three interviewers (Interviewer 1, Interviewer 2 and Interviewer 3) are $B_1 = \{b_1, b_2, b_3, b_5\}$, $B_2 = \{b_1, b_4, b_5, b_6\}$ and $B_3 = \{b_2, b_4, b_5, b_7\} \subseteq E$ respectively.

The tabular representation of FS$\beta$CS $(N_1, B_1)$, $(N_2, B_2)$ and $(N_3, B_3)$ are shown in Table 3, 4 and 5 respectively.

*Step 2:* The tabular representation of $\beta$-level soft sets $L_1((N_1, B_1),0.5)$, $L_2((N_2, B_2),0.5)$ and $L_3((N_3, B_3),0.5)$ are shown in Table 6, 7 and 8 respectively.

**Table 3.** Tabular representation for $(N_1, B_1)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | 0.5 | 0.2 | 0.5 | 0 | 0.6 | 0 | 0 |
| $v_2$ | 0.3 | 0.9 | 0.7 | 0 | 0.7 | 0 | 0 |
| $v_3$ | 0.7 | 0.3 | 0.8 | 0 | 0.6 | 0 | 0 |
| $v_4$ | 0.2 | 0.5 | 0.6 | 0 | 0.1 | 0 | 0 |
| $v_5$ | 0.9 | 0.4 | 0.8 | 0 | 0.5 | 0 | 0 |
| $v_6$ | 0.4 | 0.6 | 0.9 | 0 | 0.2 | 0 | 0 |

**Table 4.** Tabular representation for $(N_2, B_2)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | 0.6 | 0 | 0 | 0.4 | 0.7 | 0.4 | 0 |
| $v_2$ | 0.5 | 0 | 0 | 0.3 | 0.2 | 0.5 | 0 |
| $v_3$ | 0.8 | 0 | 0 | 0.1 | 0.1 | 0.7 | 0 |
| $v_4$ | 0.3 | 0 | 0 | 0.7 | 0.5 | 0.1 | 0 |
| $v_5$ | 0.1 | 0 | 0 | 0.6 | 0.8 | 0.2 | 0 |
| $v_6$ | 0.4 | 0 | 0 | 0.4 | 0.6 | 0.3 | 0 |

**Table 5.** Tabular representation for $(N_3, B_3)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | 0 | 0.7 | 0 | 0.1 | 0.5 | 0 | 0.5 |
| $v_2$ | 0 | 0.2 | 0 | 0.6 | 0.7 | 0 | 0.7 |
| $v_3$ | 0 | 0.6 | 0 | 0.3 | 0.6 | 0 | 0.2 |
| $v_4$ | 0 | 0.1 | 0 | 0.8 | 0.4 | 0 | 0.5 |
| $v_5$ | 0 | 0.4 | 0 | 0.9 | 0.7 | 0 | 0.8 |
| $v_6$ | 0 | 0.5 | 0 | 0.4 | 0.8 | 0 | 0.6 |

**Table 6.** Tabular representation for $\beta$-level soft set $L_1((N_1, B_1),0.5)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| $v_2$ | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| $v_3$ | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| $v_4$ | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| $v_5$ | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| $v_6$ | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

**Table 7.** Tabular representation for $\beta$-level soft set $L_2((N_2,B_2),0.5)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| $v_2$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| $v_3$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| $v_4$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $v_5$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $v_6$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

**Table 8.** Tabular representation for $\beta$-level soft set $L_3((N_3,B_3),0.5)$

| | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|---|---|---|---|---|---|---|---|
| $v_1$ | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| $v_2$ | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| $v_3$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| $v_4$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $v_5$ | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| $v_6$ | 0 | 1 | 0 | 0 | 1 | 0 | 1 |

*Step 3:* The fuzzy soft $\beta$-adhesion for $(N_1,B_1)$ are $SA_\beta(v_1)=\{v_1,v_3,v_5\}$, $SA_\beta(v_2)=\{v_2\}$, $SA_\beta(v_3)=\{v_1,v_3,v_5\}$, $SA_\beta(v_4)=\{v_4,v_6\}$, $SA_\beta(v_5)=\{v_1,v_3,v_5\}$ and $SA_\beta(v_6)=\{v_4,v_6\}$.

The fuzzy soft $\beta$-adhesion for $(N_2,B_2)$ are $SA_\beta(v_1)=\{v_1\}$, $SA_\beta(v_2)=\{v_2,v_3\}$, $SA_\beta(v_3)=\{v_2,v_3\}$, $SA_\beta(v_4)=\{v_4,v_5,v_6\}$, $SA_\beta(v_5)=\{v_4,v_5,v_6\}$ and $SA_\beta(v_6)=\{v_4,v_5,v_6\}$.

The fuzzy soft $\beta$-adhesion for $(N_3,B_3)$ are $SA_\beta(v_1)=\{v_1,v_6\}$, $SA_\beta(v_2)=\{v_2,v_5\}$, $SA_\beta(v_3)=\{v_3\}$, $SA_\beta(v_4)=\{v_4\}$, $SA_\beta(v_5)=\{v_2,v_5\}$ and $SA_\beta(v_6)=\{v_1,v_6\}$.

Let $M=\{v_1,v_3,v_5,v_6\}$, $O=\{v_2,v_3,v_6\}$ and $P=\{v_1,v_2,v_3,v_4\}$ are the subsets of candidates selected by the three interviewers respectively.

For $M$, $\underline{FS}(M)=\{v_1,v_3,v_5\}$, $\overline{FS}(M)=\{v_1,v_3,v_4,v_5,v_6\}$, $Pos(M)=\{v_1,v_3,v_5\}$, $Neg(M)=\{v_2\}$ and $Bnd(M)=\{v_4,v_6\}$.

For $O$, $\underline{FS}(O)=\{v_2,v_3\}$, $\overline{FS}(O)=\{v_2,v_3,v_4,v_5,v_6\}$, $Pos(O)=\{v_2,v_3\}$, $Neg(O)=\{v_1\}$ and $Bnd(O)=\{v_4,v_5,v_6\}$.

For $P$, $\underline{FS}(P)=\{v_3,v_4\}$, $\overline{FS}(P)=\{v_1,v_2,v_3,v_4,v_5,v_6\}$, $Pos(P)=\{v_3,v_4\}$, $Neg(P)=\emptyset$ and $Bnd(P)=\{v_1,v_2,v_5,v_6\}$.

*Step 4:* The Z-GFS$\beta$CR matrices for $(N_1,B_1)$, $(N_2,B_2)$ and $(N_3,B_3)$ are

$$[m_{ij}]=\begin{pmatrix}1&1&1&1&1&1&1\\0&0&0&0&0&0&0\\1&1&1&1&1&1&1\\a&a&a&a&a&a&a\\1&1&1&1&1&1&1\\a&a&a&a&a&a&a\end{pmatrix},$$

$$[n_{ij}]=\begin{pmatrix}0&0&0&0&0&0&0\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\a&a&a&a&a&a&a\\a&a&a&a&a&a&a\\a&a&a&a&a&a&a\end{pmatrix},$$

$$[k_{ij}] = \begin{pmatrix} a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \end{pmatrix}.$$

*Step 5:* By using Saaty's (2008) nine-point scale, the comparison matrices for each criterion according to the three interviewers are

$$B_1 = \begin{pmatrix} 1 & 5 & 2 & 1 & 4 & 1 & 4 \\ 1/5 & 1 & 1/4 & 1/5 & 1/2 & 1/3 & 1/6 \\ 1/2 & 4 & 1 & 2 & 3 & 1 & 2 \\ 1 & 5 & 1/2 & 1 & 4 & 2 & 4 \\ 1/4 & 2 & 1/3 & 1/4 & 1 & 1/2 & 1/5 \\ 1 & 3 & 1 & 1/2 & 2 & 1 & 1 \\ 1/4 & 6 & 1/2 & 1/4 & 5 & 1 & 1 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 1 & 1/2 & 3 & 5 & 2 & 3 & 2 \\ 2 & 1 & 2 & 3 & 2 & 3 & 6 \\ 1/3 & 1/2 & 1 & 4 & 1 & 5 & 3 \\ 1/5 & 1/3 & 1/4 & 1 & 1/3 & 1/2 & 1 \\ 1/2 & 1/2 & 1 & 3 & 1 & 2 & 4 \\ 1/3 & 1/3 & 1/5 & 2 & 1/2 & 1 & 3 \\ 1/2 & 1/6 & 1/3 & 1 & 1/4 & 1/3 & 1 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} 1 & 3 & 2 & 7 & 3 & 5 & 6 \\ 1/3 & 1 & 1/2 & 2 & 1 & 3 & 4 \\ 1/2 & 2 & 1 & 3 & 1/2 & 1 & 2 \\ 1/7 & 1/2 & 1/3 & 1 & 1/4 & 1/2 & 1/5 \\ 1/3 & 1 & 2 & 4 & 1 & 2 & 3 \\ 1/5 & 1/3 & 1 & 2 & 1/2 & 1 & 1/2 \\ 1/6 & 1/4 & 1/2 & 5 & 1/3 & 2 & 1 \end{pmatrix}.$$

By using AHP method, the weights for each criteria are calculated and it is attained as

$$[W_{B_1}] = \begin{pmatrix} 0.244 \\ 0.035 \\ 0.189 \\ 0.224 \\ 0.052 \\ 0.133 \\ 0.123 \end{pmatrix}, \ [W_{B_2}] = \begin{pmatrix} 0.233 \\ 0.274 \\ 0.17 \\ 0.049 \\ 0.143 \\ 0.08 \\ 0.051 \end{pmatrix} \text{ and } [W_{B_3}] = \begin{pmatrix} 0.355 \\ 0.148 \\ 0.138 \\ 0.04 \\ 0.165 \\ 0.07 \\ 0.084 \end{pmatrix}.$$

*Step 6:* Multiply the weight of each parameters with the corresponding Z-GFS$\beta$CR matrix. Let a = 0.6.

$[m_{ij}] \times [W_{B_1}] = [D_{B_1}]$

$$= \begin{pmatrix} 0.244 & 0.035 & 0.189 & 0.224 & 0.052 & 0.133 & 0.123 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.244 & 0.035 & 0.189 & 0.224 & 0.052 & 0.133 & 0.123 \\ 0.1464 & 0.021 & 0.1134 & 0.1344 & 0.0312 & 0.0798 & 0.0738 \\ 0.244 & 0.035 & 0.189 & 0.224 & 0.052 & 0.133 & 0.123 \\ 0.1464 & 0.021 & 0.1134 & 0.1344 & 0.0312 & 0.0798 & 0.0738 \end{pmatrix},$$

$[n_{ij}] \times [W_{B_2}] = [D_{B_2}]$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.233 & 0.274 & 0.17 & 0.049 & 0.143 & 0.08 & 0.051 \\ 0.233 & 0.274 & 0.17 & 0.049 & 0.143 & 0.08 & 0.051 \\ 0.1398 & 0.1644 & 0.102 & 0.0294 & 0.0858 & 0.048 & 0.0306 \\ 0.1398 & 0.1644 & 0.102 & 0.0294 & 0.0858 & 0.048 & 0.0306 \\ 0.1398 & 0.1644 & 0.102 & 0.0294 & 0.0858 & 0.048 & 0.0306 \end{pmatrix},$$

$[k_{ij}] \times [W_{B_3}] = [D_{B_3}]$

$$= \begin{pmatrix} 0.213 & 0.088 & 0.0828 & 0.024 & 0.099 & 0.042 & 0.0504 \\ 0.213 & 0.088 & 0.0828 & 0.024 & 0.099 & 0.042 & 0.0504 \\ 0.355 & 0.148 & 0.138 & 0.04 & 0.165 & 0.07 & 0.084 \\ 0.355 & 0.148 & 0.138 & 0.04 & 0.165 & 0.07 & 0.084 \\ 0.213 & 0.088 & 0.0828 & 0.024 & 0.099 & 0.042 & 0.0504 \\ 0.213 & 0.088 & 0.0828 & 0.024 & 0.099 & 0.042 & 0.0504 \end{pmatrix}.$$

*Step 7:* By means of max-min technique,

$[D_{B_1}] \wedge [D_{B_2}] \wedge [D_{B_3}] = Max\left\{Min\{[D_{B_1}], [D_{B_2}], [D_{B_3}]\}\right\}$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.233 & 0.035 & 0.138 & 0.04 & 0.052 & 0.07 & 0.051 \\ 0.1398 & 0.021 & 0.102 & 0.0294 & 0.0312 & 0.048 & 0.0306 \\ 0.1398 & 0.035 & 0.082 & 0.024 & 0.052 & 0.042 & 0.0504 \\ 0.1398 & 0.021 & 0.0828 & 0.024 & 0.0312 & 0.042 & 0.0306 \end{pmatrix}$$

Hence, $Max\left\{Min\{[D_{B_1}], [D_{B_2}], [D_{B_3}]\}\right\} = \{v_1\}$. Thus, the best choice of three interviewers are $\{v_1\}$. So, the candidate $\{v_1\}$ gets the job.

### 4.4. Comparative analysis

In this section, we compare Z-GFS$\beta$CR matrices with GFSR matrices developed in (Muthukumar and Krishnan, 2018) to demonstrate the importance of our model in decision making method. Both the Z-GFS$\beta$CR matrices and GFSR matrices are applied to real-life problems of finding the best alternative from the set of candidates applied for the job interview. Using GFSR matrices, we obtain $\{v_1\}$ as the best alternative. Similarly, by applying our model Z-GFS$\beta$CR matrices, we obtain the same $\{v_1\}$ as the best alternative. From the analysis, we can say that our model is effective and feasible.

## 5. Conclusion

In our work, we have defined Z-FS$\beta$CRS with respect to $\beta$-level soft set. The fuzzy soft $\beta$-covering lower and upper approximations of Z-FS$\beta$CRS satisfy the properties of Pawlak's rough approximations showing that the proposed Z-FS$\beta$CRS is significant. A new type of matrix called Z-FS$\beta$CR matrix is introduced and we redefined the concept of Z-FS$\beta$CR matrix by generalizing it. Each new definition is illustrated with examples for better understanding. Several algebraic properties and De Morgan's laws are investigated based on the study of Z-GFS$\beta$CR matrices. A novel MAGDM model is developed using the Z-GFS$\beta$CR matrices to recruit the best applicant for the assistant professor job. Using the proposed MAGDM model, we found that the candidate $\{v_1\}$ is the suitable one. Our MAGDM algorithm can be applied to any real-world problem which will give effective results. In future work, a generalized intuitionistic fuzzy soft rough matrix could be explored to develop a novel MAGDM model.

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